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SUMMARY OF PhD THESIS

DETERMINISTIC CHAOS AND FRACTALS IN COMPLEX NETWORKS MODELING

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Chapter 1.

Introduction

Contemporary events that appear to be random can be characterized by deterministic chaos using high-performance computing systems that use specific algorithms.

The discovery of fractal forms, but especially of the various ways of their application in almost all branches of science, led to obtaining results expected with great interest by specialists in each field, who were deprived of the possibility of working with exact results, without the need by imagining additional causes.

1.1. Doctoral thesis presentation

The field studied by this thesis denotes topicality and interest on the part of the scientific society, being necessary a continuous optimization required by the ever-changing nature and which offers for analysis new phenomena in almost all its branches.

Starting from several studies that at first seemed unable to provide the answers sought by researchers, today, with the help of the IT&C community, scientists can explain phenomena and behaviors of nature to combat or speculate.

1.2. Doctoral thesis purpose

Given the fact that the in-depth field analyzes complex phenomena, the purpose of this paper is to propose methods of data processing provided by various sources in order to improve the results. Moreover, capitalizing on niche knowledge, new solutions can be developed to serve the needs of contemporary society and can be a starting point for complex systems of the future.

The availability of processing elements (hardware and software) of information, stored in digital format is, at least in the last decade, vast and provides diversified solutions to find answers to questions that bother specialists engaged in the study of deterministic chaos.

Therefore, all that remains is for IT&C specialists to work closely with the scientific community in modeling complex systems for processing and analyzing deterministic chaos in order to gain new perspectives on how to approach the environment and its effects.

1.3. Doctoral thesis content

Chapter 1, introductory section, justifies the application of the notions developed by chaos theory in modeling the complex systems and networks with which we currently interact.

The second chapter contains notions from the literature in order to explain the principles that formed the basis of the studies presented later in this paper.

Chapter 3 contains a detailed study of how the elements of fractal theory can be applied in the study of several vital systems of the human body, such as respiratory, nervous or visual, in order to provide accurate diagnoses or effective monitoring of diseases.

Chapter 4 presents a comparison of two image segmentation algorithms to identify human brain diseases. The possibility of integrating this solution with those already in production indicates that although it seems a complicated method, it is a simple, ingenious and feasible method.

Chapter 5 emphasizes the need for contemporary society to have a new distance communication technology that guarantees the instantaneous availability of information regardless of its quantity. As a result, a step forward in the implementation of such a technology is represented by the antennas used. It is known that antennas can be described by fractal shapes. The many advantages of fractals give fractal antennas properties that qualify them to be part of complex, integrated and reliable communication networks and systems.

In the last chapter of this thesis, Conclusions, the principles, results and personal contributions presented in the other chapters of this paper are presented in a concise form. Also, the section includes the list of scientific papers published during the doctoral internship as well as the perspectives for further development of the studied field.

Keywords: deterministic chaos, fractals, fractal analysis, box-counting, k-means, fuzzy c-menas, lacunarity, image segmentation, fractal antennas, 5G.

Chapter 2.

Fractal theory

2.1. Fractals and fractal size

Over time, classical geometry began to have a number of disadvantages that involved trade-offs in the accuracy of recording reality. Areas such as chemistry, medicine or mathematics, but also others, needed an increased resolution to provide the best results.

The dimensions of an object characterize how it integrates into space and how it can be measured.

Euclidean geometry reached a limit of performance when its straight lines along with smooth surfaces could not characterize objects with complicated, atypical, shape. For this reason Benoit Mandelbrot (1924 - 2010), the Swedish mathematician of French origin, defined fractal geometry by publishing the reference work "Fractal geometry of nature". This new concept has great potential when it comes to the analysis of new topics by mathematicians or the development of new virtual environments by programmers [1].

A fractal assumes that parts of an ensemble are similar to the whole. Moreover, if the fractal consists of a finite number of copies, each with its own scale, easy to modify, then the fractal is called autosimilar. It should be noted that many of the natural forms are self-similar (eg fern leaf) [1].

As a result, a better understanding of the environment and the processes it undertakes requires a change in the techniques of analysis of shapes and objects and the deepening of new methods for society to discover answers to phenomena that currently seem enigmatic.

2.2. Iterated function systems and attractors

Considering (X, d) a symmetric space we will call a shape transformation:

$$f(x, y) = (a * x + b * y + e, c * x + d * y + f) \qquad (2.1)$$

where $x, y \in \mathbb{X}$ and $a, b, c, d, e, f \in \mathbb{R}$, an affine transformation in $\mathbb{R}^2$ [2][3].

The affine transformation can be described in a matrix form as follows:

$$f(x,y) = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e \\ f \end{pmatrix} \tag{2.2}$$

The changes suffered by the whole to obtain the new form can be described by means of projective transformations which are defined as a series of combinations of resizing, translations and rotations of space, if the parameters a, b, c, d, e and f of the transformation are defined as :

- $a = r * \cos \alpha$,
- $b = -s * \sin \beta$,
- $c = r * \sin \alpha$,
- $d = s * \cos \beta$,
    - r – horizontal homothesis factor,
    - s – verticalhomothesis factor,
    - α – the angle of rotation with respect to Ox,
    - β – the angle of rotation with respect to Oy,
- e – horizontal translation,
- f – vertical translation.

Therefore, any fractal form that has self-similarity is associated with an affine transformation that defines the changes made to the whole in order to obtain the component in question.

The literature states that fractal forms can be defined as a set of contractile affine transformations from $\mathbb{X}$: $(f_i)_{i=1...n}$, called Iterated Function System.

Contractual transformation denotes a transformation f of a metric space (X, d) such that $d(f(x)f(y)) < s * d(x, y)$ whatever the points $x, y \in \mathbb{X}$, where $s \in (0,1)$ is called the scale factor (or contraction factor).

If through a system of iterated functions $(f_i)_{i=1...n}$ we pass an image, through the feedback method, we will notice after each iteration that the resulting image is the same, called the attractor of the Iterate Function System (Collage Theorem). It should be emphasized that the attractor is not influenced by the original image, but only by the transformations made.

This concept was developed by Yuval Fisher, who made an atypical copier composed of several lenses that change the size of an original photo, and then superimpose several copies of it. This device operates in feedback mode and initiates a procedure for inserting the image obtained after a processing cycle into the system.

2.3. The direct hypothesis and the inverse hypothesis

The characterization of fractal objects with the help of any finite set of affine transformations leads to the appearance of two dual hypotheses: the direct one and the indirect one.

The direct hypothesis describes the creation of the fractal starting from the defining parameters of the set of affine transformations. This method is the basis for the generation of many types of established fractals, such as: Barnsley's fern leaf, Koch's curve, Sierpinski's triangle, etc. The process involves the following steps:

- We start from a certain point (x, y), and after applying one of the affine transformations in the set (chosen randomly) to the point (x, y), we obtain a point (x_1, y_1).
- Repeat the first step, but consider (x_1, y_1) as initial point.

The inverse hypothesis is the procedure that constitutes fractal compression, being initiated by Michael Barnsley. He proposed storing the parameters at the expense of the image. The advantage is the occupation of a smaller memory area, and the disadvantage is the determination of the parameters of the iterated function system, which is a bit more complicated.

2.4. Fractal size

An object may be perceived differently depending on the size used during the measurement.

The topological dimension is closely correlated with the characteristics of the points that make up the object to be analyzed and states that the point has dimension 0, the line or curve is dimension 1, the surfaces have dimension 2, the bodies have dimension 3 and so on, not considering the dimension with a unit. the larger the space in which the listed items are sunk. Thus, the topological dimension is limited to homomorphic transformations. Therefore:

- $d_T = 0$, if the object to be analyzed is not connected (isolated point).
- $d_T = k, k \geq 1$, if any point belonging to the object to be analyzed has a neighborhood V (p) whose edges have the dimension $d_T = k - 1$.

Taking as an example the points belonging to a narrow curve (k = 2), it is deduced that they have neighborhoods in the form of segments whose edges are two points of size $d_T = k - 1 = 1$ [1][2][3][4][5].

Therefore, we find that a curve sinks into a Euclidean space with size equal to 3 even if it has topological size 2.

With the emergence of the notion of fractal geometry, the characterization of a shape through the prism of topological dimensions, expressed by an integer, proved to be insufficient. An example that can confirm this is Koch's curve. This shape describes an interesting phenomenon with an area equal to 0 and an infinite perimeter (after each iteration its length increases 4/3 times). Thus, from the point of view of Euclidean geometry, Koch's curve is an object of magnitude 1, with infinite perimeter, but it cannot be said that it is also two-dimensional because its area is zero [1].

Therefore, the notion of fractal dimension is introduced, whose value is expressed by a rational number. In addition, the very notion of fractal is closely related to that of the fractal dimension. A fractal shape is a figure whose fractal size is strictly larger than the topological dimension [5].

2.5. Hausdorff dimension

The dependence of the scale used makes fractal objects difficult to measure in the context of classical geometry. Their physical properties (length, surface, volume) depend on the representation of the resolution.

Around 1914, the German mathematician Felix Hausdorff proposed a new concept of topological spaces, claiming that the fractal size is proportional to the minimum number of spheres, of given radius, needed to cover the measured object [5].

Today, to facilitate computer processing, cubes or rectangular (possibly square) surfaces are used.

For example, to cover a curve of length 1, they are required $N(s) = \frac{1}{s}$ squares with side s. To cover an area with area equal to 1, are required $N(s) = \frac{1}{s^2}$ squares of side s, and to cover a body with a volume equal to 1, are required $N(s) = \frac{1}{s^3}$ squares of side s. Usually the relation is checked:

$$N(s) \sim \frac{1}{s^D} \tag{2.3}$$

unde $N(s)$ reprezintă numărul de pătrate necesare, s definește latura unui pătrat, iar D reprezintă dimensiunea formei [5].

Dacă se logaritmează relația (2.3) se definește D ca:

$$D \sim \frac{\log N(s)}{\log \frac{1}{s}} \tag{2.4}$$

Thus, the Hausdorff dimension, also known as the Hausdorff-Besicovich dimension, is defined as follows:

Let $d, s \in \mathbb{R}$ și $N(s) = f(d) * s^d$ a set of functions, so that N (s) is the number of spheres of diameter s necessary to cover the given set F. Then we find that there is only one value $d = D_H$, called the Hausdorff dimension of F, so that [5]:

$$d < D_H \Rightarrow N(s) \longrightarrow \infty$$
$$d > D_H \Rightarrow N(s) \longrightarrow 0 \qquad (2.5)$$

An object characterized by the Hausdorff dimension also benefits from a measure given by the relation:

$$N(s) * s^{D_H} \qquad (2.6)$$

The Hausdorff concept of size is recognized as the most widely accepted definition of fractal size. Instead, it has the disadvantage of being difficult to calculate.

Usually, the fractal dimension is estimated using the self-similar dimension or the box-counting dimension.

Fractals were originally defined by Mandelbrot as shapes with infinite details at any scale. Later, the mathematician reformulated the definition by issuing the notion of self-similarity, arguing that fractals are shapes made up of parts similar to the whole. He then made a second reformulation stating that fractals are the shapes that meet the condition that their Hausdorff size be strictly larger than the topological dimension [5].

2.6. Autosimilar fractal dimension

It is well known that similarity is found among the fundamental features of fractal objects. Considering Mandelbrot's statement that a coastline resembles a straight line from a distance, and as the observation distance from it decreases, it resembles a broken line, we find that we can classify the similarity according to the scale [1][2][3]:

- self-similarity - the fractal consists of its copies at different scales of representation. Most often it corresponds to computer-generated fractal shapes, having applicability in fractal compression processes.
- sectoral similarity - the fractal is composed of children close in appearance. Both artificial and natural fractals can fall into this category.
- Brownian - the fractal is divided into random fragments, with details at each scale. Among the most common fractal forms with this property we mention the coastlines and plasma.

Artificial fractals, increasingly used in modeling seemingly random phenomena, are the most studied category of fractals under study by fractal geometry due to self-similarity. Having as a starting point the purpose of the fractal dimension, ie the measurement of the level of fragmentation of an object, in terms of self-similar fractal shapes can be reinterpreted the following relation [1]:

$$D_F = \frac{\log number\ of\ self-similar\ copies}{\log contraction\ factor} \quad (2.7)$$

Taking as an example the Sierpinski triangle, where we have three copies of the original form and a topological dimension equal to 2, we find that the fractal dimension, according to relation (2.7) will be 1.5849. Therefore, the self-similar dimension evaluates the scale invariance of the shape at affine transformations. The notion of self-affinity is also found in the literature, which indicates a statistical invariance of scale.

2.7. Box-counting algorithm

The advent of the computer led to the automated calculation of the fractal size when analyzing binary images, stored as pixel arrays.

Covering the image to be analyzed with side squares leads to:

$$D = \lim_{s \to 0} \frac{\log N(s)}{\log \frac{1}{s}} \quad (2.8)$$

Assuming that this limit exists, it will be called the box-counting dimension of the analyzed shape. Given that this type of boundary converges in a slow manner, it is necessary to use an alternative method.

Given the fact that $\log N(s) = D * \log \frac{1}{s}$ represents the equation of variable s of a slope line D, thus the log-log curve can be realized, defined by the points $\left(\log N(s), \log\left(\frac{1}{s}\right)\right)$, following that by means of the linear regression the slope of the curve can be deduced, in this case the fractal dimension [1][2][3][6]:

$$D_f = \frac{n^2 \sum_{i=1...n} x_i y_i - \sum_{i=1...n} x_i * \sum_{i=1...n} y_i}{n^2 \sum_{i=1...n} x_i^2 - \left(\sum_{i=1...n} x_i\right)^2} \quad (2.9)$$

where $x_i = \log \frac{1}{s}$ and $y_i = \log N(s)$ for various values of s.

The box-counting method has as utility the calculation of the fractal dimension taking into account the variation of the object size in relation to the chosen scale factor. The procedure involves successively dividing the image into 4, 16, 64 and so on equal borders and the quantization of the borders covering the object at each iteration.

Coordinate points $\left(\log N(s), \log\left(\frac{1}{s}\right)\right)$ they will be arranged approximately on a straight line with a slope equal to the box-counting size [1][2][3][6][7].

Chapter 3.

Analysis of complex networks in human body

Throughout history, parts of the human body have been analyzed by several techniques either to confirm proper functioning or as a result of symptoms. Today, in order to give the specialist an overview of the structure, functioning, but also of the history of an organ, computerized techniques are used that have implemented specific algorithms formaking and processing medical images.

The systems that make up the human body (eg nervous system, circulatory system, etc.) can be viewed as networks and studied using fractal analysis algorithms. Thus, the history of diseases, but also predictions about their evolution are easier to make.

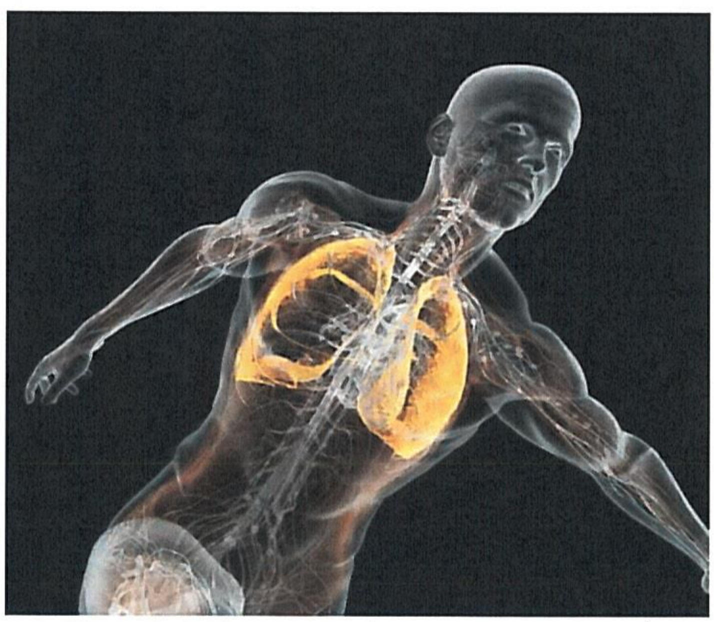

Figure 3. 1 The complexity of the human body [1]

[1] https://listverse.com/2012/09/01/10-more-amazing-facts-about-our-bodies/, website accessed on August 12th 2020, 14:51.

3.1. Lungs - anatomy and fractal analysis

3.1.1. Lung anatomy

The literature defines the lung as a paired organ, conical in shape, protected by the visceral pleura and located in the thoracic cavity, on the sides of the heart [8].

Through them the body exchanges gas between the circulatory system and the outside of the human body.

Depending on the age of the individual, but also other factors (smoking, pollution, etc.) the lungs can acquire different colors as follows: brown in the fetus, pink in the child and gray in the adult.

The main components of the lungs are the bronchial tree, lobules, branches of the lung vessels, other elements of connective tissue.

3.1.2. Fractal analysis of the lungs

In order for an X-ray to be used as input data in the fractal analysis process, a preliminary filtering is required in order to eliminate the noise introduced by the image capture equipment, but also to eliminate the texture of the lung which has a luminance close to that of the arteries.

In order for the image processing to be qualitative and for the binarization not to introduce errors, it is desirable that the background of the image be uniform in terms of luminance. To perform this procedure, you need to adjust the background as a separate image and then subtract the background as a separate image [9].

Creating the adjusted background image requires that all pixels representing the texture of the lungs be removed from the image using a morphological aperture. This type of operation has the effect of removing isolated objects or those that do not belong to continuous surfaces and which have the potential effect of altering the results.

Once the arteries are removed from the x-ray, the result can be subjected to fractal analysis. In this case the box-counting algorithm can be applied to determine the fractal properties of the extracted arteries.

Assuming there is a fractal set C with the size $D_F < D$, then the number N of squares of side R required to cover the whole set is given by the relation R^{D_F}. D_F is known in the literature as the Minkowski-Bouligand dimension, or the Kolmogorov dimension, or, more simply, the box-counting dimension.

Calling the form algorithm $[N\ R] = boxcount(C)$, where C is a matrix with D dimensions ($D = 1,2,3$), will lead to the determination of the number N of the square of side R necessary to cover the non-zero elements of the analyzed set. The cassettes used were defined with the power side of 2 (e.g, $R = 1,2,4 \dots 2^P$, where P represents the smallest integer such that the largest dimension in C is less than or at most equal to 2^P). If the length of C on each dimension is less than 2^P , C it is completed with null elements until its size reaches 2^P on each dimension. Thus, the returned vectors, N and R are of length P + 1. In the case of RGB images, a summation of the three RGB planes is recommended at the beginning of the fractal analysis process [10].

The process of filtering tissues that are not of interest can result in parasitic pixels. They can be removed by binarizing the image by specifying a threshold above which pixels with a luminance greater than this threshold will become 1, and the others 0.

Additionally, the boxcount function allows you to specify a parameter, slope, to display the semi-log graph of the local slope $D_F = -\frac{d \ln N}{d \ln R}$ as a function of R. If D_F is constant along an interval R, then D_F is the fractal dimension for the set C.

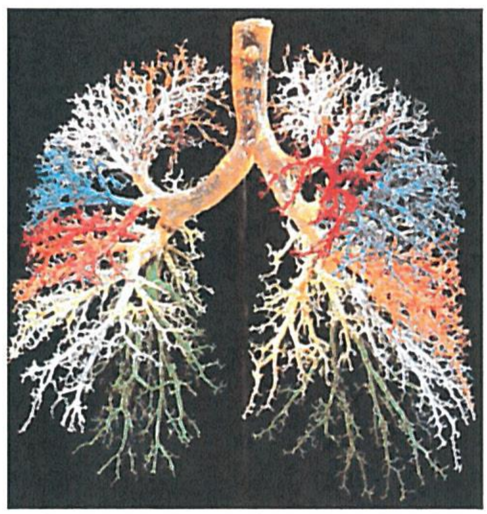

Figure 3. 2 Fractal arrangement of the elements in the human lung [2]

3.1.3. Optimizations to computerized fractal analysis techniques

Following the deepening of the literature specific to fractal analysis, but also the detailed study of the operation of the box-counting algorithm, it is found that it can be optimized by using a variable size box in order to obtain a fractal size as close to reality. Thus, for the images studied in this section, it was possible to obtain a smaller standard deviation by using a set of five masks with variable size in the form of a matrix with five rows and two columns. Specifically, the dimensions of each mask (rx and ry) were

[2] https://www.researchgate.net/figure/Resin-cast-of-human-lung-Complicated-dichotomously-branching-tree-of-conducting-airways_fig1_330666802, website accessed on August 11th 2020, 11:49.

{1,2},{2,4},{4,8},{8,16} and {16,32} pixels. The boxes went through the image successively similar to the operation of the classic box-counting algorithm.

Due to the difference between the length and width of each mask, the display of the results was made in a 3D graph for whose axes have $\log\frac{1}{r_x}$ on x axis, $\log\frac{1}{r_y}$ on y axis și $\log(n)$ on z axis.

The determination of the fractal size was performed by an equation of form $z = f(x,y)$, where $x = \log\frac{1}{r_x}$, y= $\log\frac{1}{r_y}$ and $f = \log n$, n being the number of D-dimensional boxes of caliber {rx, ry} used to cover non-zero elements of the binary image.

Therefore, the equation used for the algorithm specific to the square mask, $f(x) = a + bx$, has been adjusted so that $f(x,y) = a + bx + cy$.

The use of the square mask involves $x = y$, which involves $f = a + (b + c)x$, resulting in the end $D = b + c$.

Moreover, if:

$$b = \frac{\partial f}{\partial x} \text{ și } c = \frac{\partial f}{\partial y} \quad (3.1)$$

$$\text{resulting } D = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \quad (3.2)$$

The validation of the algorithm optimization was done by calculating the fractal size for the same data set through a dedicated program, Harmonic and Fractal Image Analyzer Demo version 5.5.30.

Moreover, the optimization of image processing can be done in the stage of filtering the useless tissue. Thus, it is desirable an algorithm that removes as much of the surplus as possible to facilitate the determination of a fractal dimension as accurate as possible, but also a coefficient of lacunarity.

Therefore, two other lung radiographs were processed to highlight the importance of separating the useful components of the information from the insignificant details.

For the grayscale version of the two analyzed images, a mask was defined, using the MATLAB R2017a software, to facilitate the separation of the lung region from the other components in the radiographs. Crop results are true to a fairly high percentage of the original image.

Then, after extracting the area to be studied, the image was transformed into a binary version in order to apply the box-counting algorithm in the classic form, but also in the version with a rectangular window.

3.2. The brain - anatomy and fractal analysis

The computational capabilities of the brain can be described as having a fractal shape due to the multiple connections between neurons and microglia (small cells with rich branches to protect neurons against pathogens and accumulate cell debris, having a phagocytic role) [11]. This type of cellular topology apparently has a chaotic aspect, but, nevertheless, through the possibilities offered by modern computer programs, medicine can evolve [7].

3.2.1. Central nervous system

The body is embedded in the environment through two systems, the nervous and the endocrine. At the same time, these systems provide the maintenance of normal levels of the internal body structure (homeostasis). Communication between two cells of the nervous system is done with the help of electrical signals, in a fast and characteristic manner, involving instantaneous responses. Regarding the endocrine system, the activity of the endocrine system is more difficult, with the help of hormones, these being chemical components accumulating information, contained in blood or lymph.

The three most important activities of the nervous system are: integration, issuing optimal responses and reception. Thanks to the large number of receptors (exteroreceptors - elements that capture information from the external environment; proprioceptors - elements that accumulate information transmitted by muscles, tendons or joints; interoreceptors - elements that capture information emitted by the viscera, also called visceroreceptors), the two environments are profiled , then, with the help of ascending pathways, on the cerebral cortex [11].

3.2.2. Neurons

Nerve impulses are actually electrochemical signals that are produced and transmitted using the capabilities of neurons. These cells differ radically from other categories of such cells:

- cannot divide (being amyototic cells)
- have the ability to function optimally throughout the entire life cycle
- have a high metabolic rate (they need glucose and oxygen all the time, dying in about three minutes if they are not fed in this way)

3.2.3. Fractal analysis of neurons

Fractal size is one of the best recommended methods for defining the neural network from the point of view of the dendritic tree. It can also provide a solid knowledge through which we can differentiate between all classes of neurons. Moreover, the fractal size has been used extensively to distinguish whether a person has a pathological condition and provides important details about its stage of development. Although it is possible for neurons and their connections to be classified by fractal dimension, it is not yet possible to establish clear relationships between fractal dimension and neurophysiological actions. This type of relationship remains in theory for the time being [12][13].

If the central nervous system were analyzed from a three-dimensional point of view, it would be observed that its way of working has fractal components. It seems that dendritic trees are the main method by which the topology is connected and a large part of the literature in the field emphasizes their fractal dimension.

The fractal size used to measure the modeled spines was the box-counting (D_B) size demonstrated to be robust and sensitive to neuronal cell morphology and other cells, including microglia. [12].

3.2.4. Fractal analysis of microglia

As in the case of neurons, over time, several papers have been published describing the use of fractal analysis in the study and classification of microglia morphology in neuroanatomy, pathology and development.

One difference between the two fields of fractal analysis of neurons and microglia is that the fractal size of microglia has been more clearly correlated with function. Using the box-counting algorithm, it has been shown that, in principle, microglia in a normal, healthy brain are strongly branched with a relatively high value of D_B and in response to certain stimuli, such as chronic stress, they can hyper-branch to a state with a D_B as large as possible, but when they respond to completely harmful pathological events, such as brain trauma, they enter a debranching cycle that involves the concomitant decrease of D_B, and then they return through a cycle of re-branching and growth of D_B when they resume their normal activity.

Much of the literature on fractal microglia analysis refers to the use of the box-counting algorithm based on binary silhouettes or contours, a strong point being that this methodology is robust for all general, branched and unbranched morphological types, facilitating thus comparisons.

Similar to what has been observed for neurons, the size of the fractal is usually positively correlated with the increase in the size of the span covered by a cell, but provides in addition to its size and information about its characteristics.

3.2.5. Lacunarity

Literally, the word "lacunarity" comes from the Latin "lacuna" and describes a gap or a hole in the integrity of a thing.

The last decades have shown an important progress in terms of fractal analysis in medicine and one of the newest notions introduced is the study of gaps. This parameter provides details about the uniformity of an object, a low value of which makes the object homogeneous, while a high value implies heterogeneity.

Regardless of how the box-counting algorithm is implemented, the degree of gaps will be calculated starting from the probability distribution of the pixels, also known as mass distribution [7].

The number of pixels per box will determine the distribution of pixels according to the size of the box or the scale (ε), a value that is inversely proportional to the size of the box. For a certain value (ε), the degree of gaps will be denoted with λ_ε, this being determined for the distribution of pixels as a coefficient of variation square, *CV* [7]:

$$\lambda_\varepsilon = (CV)^2 = \left(\frac{\sigma}{\mu}\right)^2 \tag{3.3}$$

where σ is the standard deviation and μ the average of the pixels per box as a function of the value ε.

3.2.6. Lacunarity measurements using the box-counting algorithm

An important aspect to consider in interpreting the lacunarity in a fractal analysis is that the fractal size and the lacunarity can be correlated. Smith, Lange, and Marks (1996), for example, found that the values they calculated for lacunarity were negatively correlated with the fractal size of the mass.But the relationship between the fractal dimension and the lacunarity is not simple. The prefactorial lacunarity, for example, depends very much on the size of the box used by the box-counting algorithm and is generally redundant with the fractal size.

In addition, the correlation between λ and the fractal dimension given by the box-counting algorithm can be positive or negative, depending on the images/models analyzed. For example, for a series of decreasing calibration grids (i.e., increasing density), λ decreases mainly as the cassette fractal size increases, but for a series of increasingly complex fractal contours, groups of gaps have been observed of different sizes, as well as the increase of the parameter Λ [12]. The overall results suggest that the gap and fractal size may be correlated in some cases, but are not necessarily redundant. Indeed, studies show that sometimes models that cannot be differentiated by their fractal dimensions are distinguished by their lacunarity or vice versa.

3.2.7. Box-counting algorithm versions used in lacunarity analysis

Grayscale method

There are some differences in the application of the box-counting algorithm to binary images and its application to grayscale images.

Detail *N*, from the fractal size equation, measured in the case of box-counting analysis of an image containing gray tones, does not depend on the number or pixels concentration, but on their intensity. This is because the restriction that limits the pixel values either in the background or in the foreground of binary images does not apply to grayscale images; rather, the pixel values are covering a range [13][15].

The fractal analysis described so far here is sometimes called monofractal analysis, in order to be distinguished from multifractal analysis, which identifies models better characterized by a spectrum of D_F than by a single D_F.

The multifract analysis process is analogous to applying warp filters to an image to exaggerate features that might otherwise be overlooked.

Mono- and non-fractals are little or not affected by distortions, but multifractals are affected in characteristic ways that are used to distinguish them from mono- and non-fractals [12].

Warp filters are a set of arbitrary exponents commonly denoted by the symbol Q. In fractal analysis software, the set Q is something that the user usually manipulates from a preset default set. A generalized dimension (D_Q) is determined for each Q, based on the mass size (D_M). *N* from the formula for the fractal dimension for D_M is the average of the probability distribution of all masses for a size, thus reflecting the image density.

For D_Q, each mass is distorted by rising to Q, then it will turn out that N_Q is the average for this distorted density distribution for a given size.

Graphs are generated based on these data, with predictable features that distinguish between non, mono, and multifractal scaling, and also quantify multifractal features. [12].

Subscanning

Another method that someone interpreting a result of fractal analysis needs to understand is subscanning analysis, used to investigate variation in space. Subscription analyzes several independent domains that may or may not overlap in a single image.

Subscription and multifractal analyzes highlight variations on a single set of information; but multifractal analysis investigates a model as a whole while subscanning investigates local areas on a single image independently of other areas and then presents the results so that the local variation of D_B can be compared and evaluated [29].

3.2.8. Application developed for lacunarity calculation

To calculate the lacunarity of a binary image, a software has been developed in the MATLAB R2017a environment that involves applying with a sliding window on the image of interest, window whose size doubles until the maximum size of the image of interest is reached.

The program starts by opening a window with which you can select the image of interest by entering its name and the format in which it is stored on your computer. Then, using the *size* function, the dimensions for the image of interest are extracted to define the iteration limits of the program.

During each iteration the window scrolls the image pixel by pixel and counts the number of pixels of value 1 inside it (variable marked with "one_pixel" in the program), as well as the square of this counter (variable marked with "one_pixel2" in the program).

3.3. The eye - anatomy and fractal analysis

3.3.1. Anatomy of the eye

The eye represents the organ of the human body responsible for sight, the most significant sense of man, it receiving daily, on the surface of the retina, a very large capacity for information necessary for life's development. Given the complexity of its structure, more than two million components, the eye ranks second in the top of the most complex organs, after the brain.

3.3.2. Eye disorders

One of the most important elements of the eyeball that deserves protection is the retina. Among the pathological conditions in which it can be found we find [33]:

- Age-related macular degeneration - usually occurs in people in old age, being the most common macular disease and has two forms:
    - Wet (exudative)
    - Dry (atrophic)
- Epi-retinal membranes - stages of the first mentioned disease that in advanced stages severely damage the macula (macular hole).
- Central retinal dystrophies.

- Inflammatory maculopathies of the types:
  - Contagious (toxoplasma, toxocara)
  - Non-contagious (central serous chorioretinopathy) - common among young people.

3.3.3. Retinal fractal analysis

The arrangement of the blood apparatus in the eyeball seems to be of the fractal type, which offers the possibility to introduce new methods for identifying diseases of the visual system.

With the help of established algorithms, such as box-counting, it can be determined whether the system responsible for eye irrigation is properly sized for the individual to benefit from a quality view.

By studying the degree of lacunarity of the fundus, it is possible to signal the appearance or monitor the evolution of a disease.

Image segmentation algorithms can have a high potential in identifying the appearance of diseases such as retinal detachment, retinopathy, pigmentation, etc.

For this study, three images were chosen that contain representations of the fundus affected or not affected by various specific diseases.

The steps of analyzing these images are similar for each case and consist of:

1. Upload an image of interest to the program.
2. Transform the selected image into grayscale.
3. Filtration of contained noise.
4. Create a mask to remove additional content.
5. Extraction of stakeholders in order to apply algorithms specific to fractal analysis.
6. Retouching the resulting image by removing parasitic pixels.
7. Image binarization.
8. Application of algorithms of interest.
9. Interpretation of results

Chapter 4.

Image segmentation algorithms to identify diseases

Thanks to increasingly advanced imaging techniques, we can today identify diseases such as stroke or Alzheimer's by using techniques such as k-means segmentation or Fuzzy c-means (FCM). Thus, these methods group the pixels of the same type into clusters which, after processing, provide an image in which the areas where the diseases take place can be easily identified.

Therefore, as long as some components of the brain, such as the blood vessel network or the neural network, have a fractal disposition, we can easily analyze their structure in order to provide the most accurate predictions or treatments for the patients concerned.

4.1. The k-means algorithm

Considering a space with several dimensions, we can define a number of groups (clusters) through which to associate points with similar characteristics.

The definition of the hypothesis in which the distances between the points of a set have values eligible for the formation of a cluster, depends on an adapted method of measuring them. Thus, let $D(x, y)$ be a measure of the distance by which we can define the following cases [35]:

- $D(x,y) = 0$. The spacing of a point to itself is 0.
- $D(x,y) = D(y,x)$. This equality implies symmetry.
- $D(x,y) \leq D(x,z) + D(z,y)$. Triangle inequality.

In most cases the points are defined in a space of k dimensions, the distance between two points $x = [x1, x2...xk]$ $și$ $y = [y1, y2...yk]$ being given by one of the following formulas [35]:

- Norm L_2 (common distance): $\sqrt{\sum_{i=1}^{k}(x_i - y_i)^2}$ (4.1)
- Norm L_1 (the Manhattan distance): $\sum_{i=1}^{k}|x_i - y_i|$ (4.2)
- Norm L_∞ : $max_{i=1}^{k}|x_i - y_i|$. (4.3)

When the definition of a Euclidean space is not possible, the grouping of points becomes a delicate matter.

The k-means algorithm keeps the information in the central memory, defining a number of k centroid and associating the points of their nearest centroid. A centroid can migrate with the assignment of points.

Unlike hierarchical clustering, k-means acts on real observations, not on each pair of observations, thus becoming more suitable for processing significant amounts of data.

In a cluster obtained with the k-means function are found the centroid (calculated using different methods for each value of the accepted distance) and the member objects, located at a distance minimized as much as possible by the algorithm. Also, the number of iterations of the algorithm can be controlled by the user [36].

4.2. The Fuzzy C-Means algorithm (FCM)

The Fuzzy C-Means technique is one of the most popular image segmentation methods. With easy implementation, this process can provide desirable results if the hardware resources of the system running on it are generous. For 2D data sets, the execution time, the required memory and the quality of the results do not require very high values. Instead, for 3D sets, significant resources are recommended in order to achieve efficient implementation.

Computational efficiency is obtained by using the image intensity histogram during the clustering process instead of the raw image data.

The elements of the FCM algorithm are:

- a set of *n* objects $X = \{x_1, x_2 \dots x_n\}$, in which x_i is a point with *d* dimensions
- a partitioning matrix $W = w_{i,j} \in [0,1]$, $i = 1 \dots n$ și $j = 1 \dots k$, in which each element $w_{i,j}$ represents a weight that gives the degree of object *i* to the cluster C_j.

The algorithm's constraints are given by the following statements:

- each weight for one point x_i must be 1.

$$\sum_{j=1}^{k} w_{i,j} = 1 \qquad (4.4)$$

- each cluster C_j contains, for a non-zero weight, at least one point, but for a weight of 1 will not contain all points.

$$0 < \sum_{i=1}^{n} w_{i,j} < n \qquad (4.5)$$

The computational process by which the centroids of a cluster C_j are calculated is governed, from a mathematical point of view, by the following relation:

$$c_j = \frac{\sum_{i=1}^{n} w_{ij}^{p} x_i}{\sum_{i=1}^{n} w_{ij}^{p}} \tag{4.6}$$

Notice that equation (4.6) is a nuanced expression of a relationship that defines a centroid using the k-means algorithm. The difference is that the points are taken into account in full and their contribution is weighted by the membership coefficient.

The update process is performed by decreasing the amount of average errors as follows:

$$w_{ij} = \frac{(1/dist(x_i,c_j)^2)^{\frac{1}{p-1}}}{\sum_{q=1}^{k}(1/dist(x_i,c_q)^2)^{\frac{1}{q-1}}} \tag{4.7}$$

4.3. Comparison of the two algorithms

Theory characterizes the FCM algorithm as similar to the k-means method, calling it Soft K-Means.

In principle, the functions are identical, the only difference being the use of a value that defines the degree of belonging of a point to each of the clusters used. This coefficient is closely related to an exponent, called "stiffness", which has the role of taking into account the stronger connections between points. It should be noted that when the stiffness value tends to infinity, the determined vector becomes a binary matrix, resulting in an identity between FCM and k-means algorithms.

Thus, mathematically speaking, we notice in the definitions of the functions that characterize the two methods a similarity in terms of minimizing the sum of the average error (SEM):

- k-means:

$$SEM = \sum_{j=1}^{k} \sum_{x \in C_i} dist(c_i, x)^2 \tag{4.8}$$

- FCM:

$$SEM = \sum_{j=1}^{k} \sum_{i=1}^{n} w_{i,j}^{p} dist(x_i, c_j)^2 \tag{4.9}$$

where p represents an index that characterizes the influence of the weight, $p \in (1,\infty)$.

From the point of view of computational performance, the FCM technique must perform k multiplications for each point and for each dimension, actions that make it slower than the k-means method. On the other hand, if a cluster is to be analyzed in which the points are dispersed along a certain size or two, the use of FCM to the detriment of k-means is recommended.

4.4. Complex characterization of cranial tomography. Case Study

The realization of this case study involved the analysis of two cranial tomographies that present two of the most common diseases of the human brain.

Starting from these images, image processing techniques were applied in order to calculate the fractal size, lacunarity and degree of group membership in order to identify and analyze the affected areas of the organ.

A significant observation that needs to be made states that the fractal size determined using the box-counting algorithm is strongly influenced by the limitations brought by the acquisition of digital images. The pixel distribution depends on how the image was taken. It is known from fractal theory that the level of detail would remain infinitely unchanged for an authentic fractal.

The program starts by opening a window through which you navigate the hard disk to the image to be analyzed (selection is recommended).

After loading in the program, over the desired image the user defines a mask using the mouse. This mask describes the area for which the fractal size and lacunarity will be calculated.

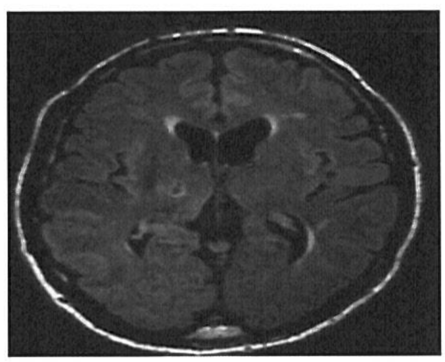

Figura 4. 1 Hyperintensities of white matter in vascular contributions to cognitive impairment and dementia [3]

[3] https://www.sciencedirect.com/science/article/pii/S2352873719300046. website accessed on June 2nd 2020, 14:34.

Chapter 5.

Applications of fractals in modern telecommunications networks

Whether we are talking about access to entertainment information or personal information, links to sources of information are not allowed, from the customer's point of view, to be delayed or unable to be accessed. At this time, each of us has a smart communications terminal through which we access applications to facilitate our daily activities.

The IT&C environment started some time ago to develop programs for most of the actions that a person can take on a daily basis. Thus, the possibility of parallelizing activities was born. Moreover, "social media" applications have developed impressively in the last decade due to the fact that they have allowed people to be interconnected and to share various information with each other. Profile studies have shown that a user of 4G technology performs twice as much information traffic as a user of another type of communication [38].

Due to the fact that mobile operators have developed infrastructures that allow the sudden development of the IT&C environment, today customers show a growing need for denser networks, high capacity traffic, high mobility, ubiquitous coverage, low latency, large number of affiliated devices and low power consumption.

Thus, in order to optimize production costs, the design of a fractal antenna that can be used for communications in different frequency bands seems to be a feasible solution.

Fractal geometric shapes can easily define antennas or antenna networks that facilitate communication over multiple frequency ranges, if they are sized correctly.

5.1. 5G technology

Engineers struggle every day to develop appropriate solutions for fast communications that meet the needs of the data traffic company. Today, the number of services required by different types of applications is incredibly high and this only happens to run a program that makes our lives easier.

A facility that is intended to be implemented by 5G technology would be the modeling of antenna beams, a method that should also depend on the equipment of the end user. Obviously, the cost for 5G infrastructure would increase due to the huge number of connected devices, which contradicts the current generations of communications, because today the networks use low transmission powers to operate.

Suddenly, the concept of "massive MIMO" became a backup solution to increase bandwidth, but not for a long time, due to the high probability of interference. So, a good antenna design is necessary to avoid this phenomenon.

Microsecond latencies are a major challenge for 5G. This feature arose from the haste in which society finds itself at the moment.

Some contemporary processors offer possibilities to achieve this goal, but the laws of physics are what limit transfers. So the sender and receiver should be close enough to perform this process, and the topology should include a large number of hardware devices.

For Romania, regarding the bandwidths used, several intervals are recommended as follows:

- 452.5 - 457.5 MHz
- 462.5 - 467.5 MHz
- 753 - 758 MHz
- 788 – 791 MHz

The use of each frequency range implies the appearance of specific behaviors regarding the propagation of electromagnetic waves. Some of the most important phenomena to study are the reflection, dispersion, diffraction or penetration of different materials. [38].

5.2. Fractal shaped antennas

To maximize the length of the antenna, it is suggested to use fractal antennas, objects that use the self-similarity property of fractals for optimal operation in multiple frequency ranges.

According to a public document written by ANCOM[4] (page 49), PPDR communications would use some bandwidth for specific links. So PPDR systems could use a single antenna for all the required bandwidths. This could represent the possibility of developing a self-reconfigurable system.

The relationship that describes the radiated power for a multi-element antenna is as follows [40]:

$$E \cdot E^{*} \sim \left(\sum_{n=1}^{N} A_n e^{i\phi_n}\right) \cdot \left(\sum_{m=1}^{N} A_n e^{i\phi_m}\right)^{*} = \sum_{n,m} (A_n A_m^{*})\, e^{i(\phi_n - \phi_m)} \qquad (5.1)$$

[4] https://www.ancom.ro/uploads/links_files/Strategia_5G_pentru_Romania.pdf, website accessed on March 14th 2020, 20:44.

A major difference between arrays with a matrix arrangement and a fractal antenna is that the order of classical antennas makes them resonate along a bandwidth and the shape of the fractal (given by the fractal elements) makes the antenna in the fractal form resonate to along several bandwidths (iteration 0 can be viewed as a classic antenna and starting with a new iteration a new bandwidth can be obtained).

5.2.1. Koch's curve

This fractal is named after Helge von Koch, the Swedish mathematician who developed this concept in 1904. The design consists in choosing a straight line of length L and dividing it into three equal segments. Then, the middle segment is replaced by two equal segments to form an equilateral triangle. Then, with each iteration, each segment will undergo the same process.

According to relations (2.3), (2.4) and (2.5) the fractal dimension of the Koch curve is equal to 1.26 due to its four component elements.

$$D = \frac{\log 4}{\log\frac{1}{3}} = 1.26 \qquad (5.2)$$

The effective length of the new shape is calculated by multiplying the size of the initial segment by $\left(\frac{4}{3}\right)^n$, where n is the number of iterations.

Similar to this shape, the fractal known as "Koch's snowflake" can develop, a contour based on the equilateral triangle. Thus, all the resulting new triangles will undergo the same process.

5.2.2. Sierpinski's gasket

This fractal form is made artificially and was developed by Waclaw Sierpinski. The process begins with an equilateral triangle from which triangles equal to a quarter of the original triangle are removed.

The fractal size of the Sierpinski gasket is equal to:

$$D = \frac{\log 3}{\log 2} = 1.585 \qquad (5.3)$$

5.3. Comparisons of fractal-shaped antennas with established-shaped ones

As mentioned before, several applications can be developed based on fractal-shaped antennas due to their compact size (this shape can be very well placed in a personal device with many types of communications), but also multiband resonance. Applications such as radars, personal mobile phones, those on the UWB branches or devices intended for the PPDR field in 5G technology can benefit from such an element that can have as a starting point any dedicated antenna (log-periodical, monopoly, dipole, patch, etc.).

By applying techniques such as penetration or bending of a well-known shape, an improvement of the antenna parameters can be seen, as if it contained additional circuits consisting of resistors and capacitors [40]. Moreover, the use of this method also implies a decrease in costs.

From the collection of fractals eligible to describe the surface of an antenna, two shapes were chosen: Koch's curve and Sierpinski's gasket.

5.3.1. Comparison between the dipole antenna and fractal antennas described by the Koch's curve

Assuming that a device should serve multiple applications, it is inconvenient for the antenna to be changed each time. Therefore, designing a suitable antenna to cover all bandwidths is the most suitable option.

From this point of view, a dipole antenna was designed in the MATLAB R2018b medium. The width of the dipole corresponds to the diameter of an equivalent cylindrical dipole fed in the center.

Then, starting from the resulting dipole antenna, two iterations were performed to generate the fractal antenna described by the Koch curve.

According to the results of the design and simulations, the objectives were achieved.

5.3.2. Comparison between the bow tie antenna and the fractal antenna described by Sierpinski's gasket

The described antennas of fractal shapes are also suitable for use in the SHF (optimal range for mobile communications) frequency range. For this study, a comparison was made between the bow tie antenna and the antenna described by the Sierpinski's gasket fractal, and the 1800 MHz and 2100 MHz bands were chosen as bands of interest.

The bow tie antenna uses two triangles as conductors, arranged like a bow tie. It may look like periodic log antennas, but they are not exactly considered periodic logs, although some industry experts believe that the bow tie antenna is a simpler version of the "tooth" periodic antenna.

Among the advantages of the bow tie antenna can be mentioned:

- Using triangles makes the band used larger.
- Using the 60° angle allows you to connect several types of signal sources.
- Weather resistant form.
- Lower weight than a regular dipole.
- Potrivită pentru comunicațiile TV de bandă largă.

As a disadvantage, it can be mentioned that biconical antennas, including the bow tie antenna, have a low transmission efficiency at the lower end of their frequency band. Therefore, a periodic log antenna is desirable. In addition, the use of a bow tie antenna is not recommended for applications for radio detection.

The comparison antennas, described by the fractal Sierpinski gasket with one and two iterations, aim to resonate at 1.8 GHz and 2.1 GHz.

Analyzing the results obtained from the simulations performed in the MATLAB R2020a environment, it can be seen that the fractal antennas give the possibility to operate in the desired frequency ranges.

5.3.3. Fractal antenna described by the closed Koch's curve

This section includes the results that will be published in the article "5G Fractal Antenna Design Based on the Koch Snowflake", authors: Mihai-Virgil Nichita, Maria-Alexandra Păun, Vladimir-Alexandru Păun and Viorel-Puiu Păun [42].

Another type of fractal shape that can describe an antenna is the closed Koch curve. It is presented as three Koch-curved fractals arranged in an equilateral triangle and has a fractal size equal to 1.2618 due to the four copies of the whole, equal to one third of the initial Euclidean size of the original object.

Let it be a set of n iterations after the execution of which we consider the following parameters L_n(the length of a side), N_n(the number of sides), l_n(the length of perimeter) și A_n(the area described by the closed Koch's curve) with the following values [42]:

$$L_n = \left(\frac{1}{3}\right)^n = 3^{-n} \tag{5.4}$$

$$N_n = 3 \cdot 4^n \tag{5.5}$$

$$l_n = N_n L_n = 3 \cdot \left(\frac{3}{4}\right)^n \tag{5.6}$$

$$A_n = A_{n-1} + \left(\frac{1}{3}\right)\left(\frac{4}{9}\right)^{n-1} A_0 \tag{5.7}$$

where A_0 represents the area of the initial triangle ($n = 0$).

The production of the closed Koch's curve can be achieved using the IFS Transformation taking into account the following relationships [42]:

$$V[A] = \cup_{n=1}^{7} v_n(A) \tag{5.8}$$

$$v_i(x, y) = \begin{bmatrix} a_{1i} & a_{2i} \\ a_{3i} & a_{4i} \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a_{5i} \\ a_{6i} \end{pmatrix} \tag{5.9}$$

The capacity size of the resulting fractal, synonymous with the fractal size, is given by the relationship [42]:

$$d_{cap} = -\lim_{n\to\infty} \frac{\ln N_n}{\ln L_n} = \lim_{n\to\infty} \frac{\ln 3 + n \ln 4}{n \ln 3} = \frac{2 \ln 2}{\ln 3} = 1.261859550 \tag{5.10}$$

Koch's Snowflake fractal can be designed either as a surface or as a closed contour. Both forms have as iteration 0 the equilateral triangle.

A possible disadvantage of the closed contour is represented by the magnetic coupling that can occur between the segments that make up the shape. Moreover, with the iterations, the thickness of the conductor (of the fractal in this case) must be diminished so that the newly formed segments do not overlap.

Chapter 6.

Conclusions

This section contains the essential topics of this thesis and provides a summary of the objectives achieved, as well as the personal contribution made. Moreover, the scientific papers published during the doctoral internship and the possibilities for further development of this study are specified.

6.1. Results achieved

Utilizing the theories contained in the literature, I obtained a series of important results in the field of medical imaging and antenna design for complex networks and communication systems.

In Chapter 2 of this paper I presented a series of information with an introductory role in the theory of chaos and fractals.

In the first section I described in detail the need to approach the concept of fractal in terms of the advantages that these forms offer.

The following two sections present the mathematical model for making fractal shapes based on established notions of the field.

The chapter also presents aspects of the notions of fractal dimension, self-similar fractal dimension and Hausdorff dimension to complete the list of terms of interest of the subject.

At the same time, we presented the purpose and mode of operation of the algorithm with the greatest notoriety in the process of calculating the fractal dimension, box-counting.

In Chapter 3, I subjected to the process of fractal analysis several vital systems of the human body (respiratory, nervous and visual). I started from the premise that the structure of some component parts of these systems are arranged in fractal form, and for a complete characterization, the classical Euclidean geometry does not show eligibility.

I chose the first organ to be analyzed the lung whose system of arteries and blood vessels can be considered fractal. Thus, I applied to ordinary lung radiographs a series of procedures in order to improve the accounts of the forms contained in order to extract only the elements of interest from it, in this case arteries and blood vessels. For a start, I managed to standardize the background luminance. Then, I increased the contrast in a predetermined interval, following that by binarizing the resulting image the rest of the tissues, such as the lung or parts of the skeleton, will be filtered so as not to alter the results. After extracting the desired elements, I applied the box-counting algorithm in the classic shape (using the square mask), but also in an adjusted shape (using the rectangular mask) to determine the fractal size. Following the comparison of the results provided by the two variants of the box-counting algorithm, but also the comparison with the data provided by the software Harmonic and Fractal Image Analyzer Demo version 5.5.30 for validation, it emerged that the variant proposed by this thesis (that of use of the rectangular mask) leads to a decrease in the standard deviation.

Moreover, I have proposed improvements on how to filter useless tissues by defining irregular masks that manually overlap only on areas of interest. This eliminates the risk of analyzing unwanted areas or filtering out useful tissues.

The second organ analyzed was the brain, for which I also made a brief description in the first part of the section.

I then continued with the presentation of the fractal analysis process of some of the most important cells in this organ. Also, in this chapter, I stressed the importance of the term lacunarity, useful in the study of brain diseases, through a theoretical description, but also practical results. Of course, we also discussed the methods of measuring the gap and the fractal analysis of the gap.

The last organ to be analyzed was the eye. After a brief description of its structure and specific diseases, I presented how to analyze retinal fractal.

Submitting several images containing representations of retinal disorders to image processing techniques applied to the lungs and brain, I was able to obtain results that help ophthalmologists in the decision-making process on medication or surgery.

Chapter 4 presents the comparison of two algorithms for image segmentation, a useful technique in determining the presence of shapes in the processed image.

The first part of the section describes the purpose of using these algorithms in the process of medical image analysis and continues with two descriptive sections of the algorithms together with the results provided by each after processing two common tomographs.

Moreover, in this chapter a conclusion of the comparison of the two algorithms according to needs and the computing power is presented. The chapter concludes with a detailed case study that presents a complex characterization of cranial tomography.

Chapter 5 exposes the need to develop contemporary communication networks in the context of serving an increasing number of users who request to benefit from impressive amounts of instant information. I described in the first part of the section the concepts of 5G communications, a technology that aims to revolutionize current topologies with equipment worthy of supporting a larger number of subscribers and who are willing to manage revolutionary transfer rates.

The second section of the chapter presents the concept of fractal antenna and some well-known examples of them. I made a comparison of consecrated-shaped antennas with fractal-shaped ones to emphasize the advantage of using the latter in several bands of interest in order to avoid coupling more radiant elements to an equipment that can use more communication technologies. The validation of the concept is presented in the graphic figures in the chapter

For the simulations and processing of this paper, several versions of the MATLAB environment were used, this programming environment being recognized for the quality of iterative analyzes and the fidelity of the simulations performed through a programming language specialized in working with matrices. The large number of functions and algorithms implemented make it possible to process any data, whether it comes from the computing unit on which the program runs or from outside it. MATLAB also includes a suite of specialized utilities, one of them, Antenna Toolbox, being used for antenna design and measurement.

6.2. Original contributions

The original contributions are briefly presented in the order in which they appear throughout the chapters of this thesis:

1. I have applied in a professional manner filters specific to image processing in order to optimize the fractal analysis process by eliminating possible sources of disturbance of results [A1][A2][B1] [C1].
2. Starting from the premise that some components of the human body are arranged in the form of a fractal, I managed to transform the information provided by a regular computer tomograph on the input date for the box-counting algorithm from the desire to optimize the decision process in case the occurrence of a disease of the organ in question [A1][A2][B1] [C1].
3. I offered a new perspective of fractal analysis by implementing the rectangular mask box-counting algorithm in the MATLAB environment in order to reduce the standard deviation [A1][A2][B1] [C1].
4. I developed complex quality studies of complex systems in the human body by developing software programs that analyzed fractal and lacunar lung radiographs, cranial tomography and graphical representations of the retina [A1][A2][B1] [C1].
5. I identified the possibility of applying the box-counting algorithm, both standard and optimized, only to the areas of interest in the image by manually defining selection masks. This avoids the analysis of useless areas that could lead to altered results [B1].
6. I managed to develop in MATLAB environment programs based on k-means and Fuzzy C-Means algorithms in order to identify through clustering techniques effects of Alzheimer's disease or sequelae of stroke [A3].

7. Moreover, I made a comparison of the two clustering algorithms according to the working speed, the available computing power of the user and the arrangement of the analysis points [A3].
8. Taking into account the future communication technology, 5G, but also the ANCOM specifications for its bands, I designed and simulated a fractal-shaped antenna that can serve all frequency ranges proposed by the Romanian state institution. The fractal shape that described the outline of the antenna was Koch's curve.
9. I also designed and simulated a fractal-shaped antenna based on a theory of fractal Koch's curve with iteration 0 equilateral triangle [D1].
10. Moreover, I designed and simulated a Sierpinski's gasket fractal antenna useful for the integration of communications in the band 1800 - 2400 MHz.

6.3. List of original papers

The scientific activity carried out during my doctoral internship was published in a series of five articles where I am the main author..

6.3.1. Scientific articles in ISI indexed publications

[A1] M-V Nichita, V-P Păun, *Fractal analysis in complex arterial network of pulmonary X-rays images*, University POLITEHNICA of Bucharest Scientific Bulletin, Series A, Vol. 80, Iss. 2,2018, ISSN 1223-7027, pp. 325- 339.

[A2] M-V Nichita, M-A Păun, V-A Păun, V-P Păun, *Fractal Analysis of Brain Glial Cells. Fractal Dimension and Lacunarity*, University Politehnica of Bucharest Scientific Bulletin, Series A-Applied Mathematics and Physics, Vol. 81, no. 1, pp. 273-284, 2019.

[A3] M-V Nichita, M-A Păun, V-A Păun, V-P Păun, *Image Clustering Algorithms to Identify Complicated Cerebral Diseases. Description and Comparison,* IEEE Access, DOI 0.1109/ACCESS.2020. 2992937.

6.3.2. Scientific articles in AIP indexed publications

[B1] M-V Nichita, M-A Păun, V-A Păun, V-P Păun, *Pulmonary X-Ray Images. A Fractal Analysis*, TIM 19 Physics Conference, Timișoara, 2019, DOI: 10.1063/5.0001033.

6.3.3. Scientific articles in Conference Proceedings

[C1] M-V Nichita, V-P Păun, P*ulmonary arterial network. A fractal analysis of xrays images*, Proceedings of the 12th Conference of the Society of Physicists of Macedonia, pp. 54-57, September 27th - 30th, Ohrid, Macedonia, ISBN 978–608–4711–08–7.

6.3.4. Scientific articles being published

[D1] M-V Nichita, M-A Păun, V-A Păun, V-P Păun, *5G Fractal Antennas Design Based on the Koch Snowflake.*

6.4. Prospects for further development

The topics addressed in this thesis are current given that they have as a starting point the desire of contemporary society to evolve by simplifying everyday life.

Skipping routine or automation of processes become standard conditions for individuals of the current decade.

IT&C specialists benefit from knowledge that can take society to a higher level of evolution and thus have to provide revolutionary solutions both hardware and software through which people can focus more on progress, personal or professional, than on routine.

Thus, among the perspectives for further development of this thesis we can mention the possibility of integrating software developed for fractal and lacunar analysis and clustering of human body components into specialized medical imaging systems in order to provide complete information to physicians.

Moreover, another benefit that can be brought to CT scanners is developing a medical image capture system capable of providing "pure" input data, without noise introduced by various specific factors, to the algorithms described in this paper.

In addition, another benefit worth considering in the fractal analysis of the components of the human body can be represented by computer systems that use biometric authentication with several factors (eg retinal analysis). In addition to the established qualities, the fractal size of a part of the human body can confirm the identity of a person.

In addition, using reliable results provided by specialized fractal analysis or clustering algorithms, new classifications of diseases or their stages of evolution can be described, thus reducing the list of inconclusive symptoms.

Also, with regard to communications, we must be aware that this field is constantly evolving in sync with the needs of individuals and that it tends to conceive of objectives that will require efforts from the scientific community.

As a result, the acquisition of current knowledge by IT&C specialists in order to develop new solutions is a condition that if not satisfied will imply the stagnation of society's progress.

Thus, whether new generations of communications will emerge or existing technologies will be integrated into hybrid systems in order to avoid possible traffic congestion generated by the large number of subscribers, new approaches to antenna design are needed.

Therefore, the proper design of emerging technologies is a task that requires professionalism and vision to design the right systems.

Regarding the fractal shaped antennas, they can benefit from improvements by designing new fractal shapes that describe antennas specific to the application in question satisfying the working conditions by capitalizing on the most important advantages of this technique of making antennas, compact arrangement, resonance in many bands of interest or the ability to implement the beam modeling technique due to the recursive property of fractals.

The complexity of radiant element simulators combined with the processing power of current computing units simplifies the testing steps and facilitates the design of antenna features that serve niche purposes.

At the same time, a study direction can be the design of high-performance RFIC, DSP and SDR modules, components of transmission and reception equipment, in order to process such frequency bands. Auxiliary equipment, such as amplifiers, preamplifiers, converters or filters, are also subsystems that should be optimized to meet new challenges.

Therefore, this thesis is based on studies of the most solid in the field of deterministic chaos and fractals and through the contributions presented extensively in previous chapters, but also in the articles I published during the doctoral internship, gives a range broad perspectives of further development with applicability in several fields of exact sciences, but not only, such as IT&C, electronics, mathematics, etc.

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