UNIVERSITY POLITEHNICA OF BUCHAREST

Faculty of Electrical Engineering

Doctoral School of Electrical Engineering

DOCTORAL THESIS SUMMARY

ANALIZA CIRCUITELOR ANALOGICE NELINIARE CU METODA HIBRIDĂ

RESEARCH ON ANALYSIS OF ANALOG CIRCUITS WITH HYBRID METHOD

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Content

Chapter 1

Introduction

The research paper entitled "**Research on analysis of analog circuits with hybrid method**" comes as a challenge for the author because he wants to develop its own specialized programs for numerical and symbolic simulation. The main objective of this thesis is represented by a complex documentation of the current state of analysis, synthesis, simulation and modeling of analog circuits, following the design, optimization of these electrical systems following simulations in specialized software. In order to optimize the analysis of electrical circuits, it is necessary to generate a mathematical model for the automatic sizing of the circuit.The long-term goal is to adapt these programs to the current needs of the engineering sciences related to the analysis of analog and digital circuits.

The paper is structured in 5 chapters as follows:

Chapter 1 - Introduction presents the general aspects related to the formulation of the analysis of nonlinear analog circuits and the objectives pursued. At present, the requirements for the analysis of analog circuits are on the one hand to develop high-performance computer programs, often adapted to a certain category of problems, and on the other hand to develop models with a certain degree of generality, which should be easy to implement and to provide results as close as possible to the real values, which would be obtained experimentally.

The objective of this thesis is the development of specialized programs using numerical and symbolic simulations as well as the simulation of circuits using these programs.

Chapter 2 entitled **Notions of nonlinear circuit analysis** presents the basics for analyzing an electrical circuit using hybrid equations. The analysis and simulation of an electrical circuit involves the formulation and solving of a system of mathematical, linear and / or nonlinear equations, which would describe its functionality.

Hybrid equations are used in the theoretical study of resistive nonlinear equations because they have an easy to analyze structure. An analysis method is all the more efficient, as it allows finding the solution (s) in a minimum time and with a minimum calculation volume.

The hybrid method, on the other hand, is more efficient as the analyzed circuit contains more linear circuit elements and controlled sources.

Chapter 3 entitled **Formulation of hybrid equations** presents examples of analysis of nonlinear electrical circuits using the method of hybrid equations. For the analysis of the entire studied analog circuit can be used, for each real solution of nonlinear hybrid equations, either the ASINOM program - Symbolic Analysis based on the Modified Nodal Method (CSAP) and the symbolic simulator MAPLE, or the ACA program - Analysis Program of Analog Circuits and the MATLAB program [11].

These programs together allow the analysis in the time domain as well as in the frequency domain of a wide range of linear, nonlinear and / or time-varying analog circuits.

In **Chapter 4** - entitled **Practical application** presents the way to solve a nonlinear resistive analog circuit with a practical application. This nonlinearity is due to the values of specific soil strengths. The solution of the nonlinear analog circuit is treated with the hybrid method, having as solution a hybrid matrix. The practical application consists in modeling the variation of the specific resistances of the soil.

The components of the practical application used consist of: IRF520 power transistor module, Arduino board, humidity sensor module, DSO138 oscilloscope kit.

In **Chapter 5 - Conclusions and Original Contributions** are summarized the results of the scientific activity carried out during the elaboration of the PhD thesis, the main original contributions made, as well as subsequent research directions.
The first attempts to solve nonlinear analog circuits were made with the Advanced Design System (ADS) design automation program. These simulations are detailed in Annexes **A1** and **A2**. Also, to understand how to simulate nonlinear analog circuits we started by simulating the Chua circuit, presented in Annex **A5**.

Dissemination of results

Dissemination of the results was achieved by publishing 9 scientific articles, of which 2 as the first author and 7 as co-author, as follows:

1. **Diana – Ramona Sănătescu**, Lucian – Vasile Ene, Mihai Iordache, *"Research on Analysis of Analog Circuits with Hybrid Method",* IEEE Xplore, 2016 International Conference on Applied and Theoretical Electricity (ICATE), DOI: 10.1109/ICATE.2016.7754703, Page(s): 1 – 7, Publisher: IEEE.
2. Ene Lucian-Vasile, **Sănătescu Diana Ramona**, *"Contributions to transfer wireless electromagnetic energy to electric cars"*, 11 Noiembrie 2016 Simpozionul de mașini electrice SME'16, SME16_paper_6.
3. Lucian-Vasile Ene, Aurel Chirilă, Dragoș Deaconu, **Diana-Ramona Sănătescu**, *"Sistem de acționare electrică pentru reglarea vitezei unei mașini asincrone trifazate de la distanță cu un PLC"*, 11 Noiembrie 2016 Simpozionul de mașini electrice SME'16, SME16_paper_7.
4. Lucian-Vasile Ene, **Diana-Ramona Sănătescu**, *"Electric Drive System for Speed Adjusting of a Three-Phase Asynchronous Motor using a PLC for Propelling an Electric Vehicle",* Proceeding of the 10^{th} International Symposium Advanced Topics in Electrical Engineering – ATEE'17, March 23-25, 2017 Bucharest, Romania, Editura Politehnica Press, ISBN: 978-1-5090-5160-1/17/$31.00 ©2017 IEEE, Publisher**:** IEEE.
5. Lucian-Vasile Ene, **Diana-Ramona Sănătescu**, Cristian Sandu, Teodor-Cătălin Bibirică, Mihai Iordache, *"Simulation of Magnetically Coupled Coils in Ansoft Q3D Extractor Program"*, Proceedings of the 13thInternational Conference on Optimization of Electrical and Electronic Equipment, OPTIM 2017, May 24-26, Brasov, România, 2017, IEEE Xplore, INSPEC Accession Number: 17028093, DOI: 10.1109/OPTIM.2017.7974971,Publisher: IEEE.
6. **Diana-Ramona Sănătescu**, Lucian-Vasile Ene, Alexandra Ionescu, Alina Oroșanu, Iordache M., *"A new approach for nonlinear analog circuits analysis"* U.P.B. Sci. Bull., Series C, Vol. 81, Iss. 3, 2019 ISSN 2286-3540, 2019.

7. Alexandra Ionescu, Alina Oroșanu, **Diana-Ramona Sănătescu**, Iordache M., *" A new approach on Chua's circuit analysis"* U.P.B. Sci. Bull., Series C, Vol. 81, Iss. 4, 2019 ISSN 2286-3540, 2019.
8. Mihai Iordache, Marilena Stanciulescu , **Diana Sanatescu**, Sorin Deleanu, Alexandra Ionescu, Anastasie Moscu , Lavinia Bobaru, „CSAP and TFSG – Circuit Symbolic Analysis Programs", SIELMEN 2019 - 12-th International Conference on Electromechanical and Power Systems10 – 11 October 2019, Chişinău Rep. MOLDOVA
9. Razvan Asanache, Mihai Iordache, **Diana Sanatescu**, Mihaela Turcu, Alexandru Grib, Lucian Ene „Wireless Charging Systems for Electrical Vehicle Batteries", ECAI-2020 (12th Edition of International Conference on Electronics, Computers and Artificial Intelligence): Nr. 5

Chapter 2

Notions regarding the analysis of nonlinear circuits

There are several types of equations that describe nonlinear resistive circuits. The nonlinear circuits are described by a several types of equations. Among them, hybrid equations [1-3] are frequent used in the theoretical study of nonlinear circuits because they have an easily analyzed structure. Hybrid equations present also the advantage of numerical analysis of circuits because they consist of a small number of variables and are separable. However, the hybrid equations are rarely used in practical applications because their formulation is quite difficult [3]. This paper considers new approaches for analysis a non-linear circuits: one using hybrid equations, Maplesoft.

Hybrid equations have an easy-to-analyze structure and have the advantage that they are separable and consist of a relatively small number of variables. Because of these points hybrid equations are applied in the theoretical study of nonlinear resistive circuits.

Analog circuits must meet certain conditions in order to formulate hybrid equations, the main one is that the circuit should allow the existence of a normal tree. Normal tree is a tree with restrictions that satisfy the following conditions in this order of priority,[8] :

a) Contains all independent ideal sources and voltage controlled

b) all nonlinear items controlled in voltage

c) contains a maximum possible number of linear resistor

d) contains no ideal independent source or controlled by current and any nonlinear element that is current controlled

e) controlled dimensions of the controlled sources are associated with linear resistors or independent variables.

The hybrid method allows in the circuit a mixture of voltage-controlled (v.c.) nonlinear capacitors, current and voltage-controlled resistors, current-controlled (c.c.) nonlinear inductor, all four types of linear controlled sources and linear inductors, capacitors, resistors, independent sources.

The algorithm is to substitute all nonlinear elements in voltage controlled (c.u) or current controlled (c.i.) by ideal independent voltage sources respective ideal independent current sources and to replace the linear capacitors and inductors by resistive discrete circuit models associated with a numerical integration algorithm (for example, the backward Euler algorithm), in this way we obtain a linear and time-invariant circuit.

The method is more efficient if the analyzed circuit includes more linear circuit elements and controlled sources.

The hybrid equation notation, [15-16]:

$$\begin{bmatrix} \boldsymbol{i}_{u,j+1}^{(k+1)} \\ \boldsymbol{u}_{i,j+1}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_{u,u} & \boldsymbol{B}_{u,i} \\ \boldsymbol{A}_{i,u} & \boldsymbol{R}_{i,i} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_{u,j+1}^{(k+1)} \\ \boldsymbol{i}_{i,j+1}^{(k+1)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{G}_{u,e} & \boldsymbol{B}_{u,j} \\ \boldsymbol{A}_{i,e} & \boldsymbol{R}_{i,j} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{j+1} \\ \boldsymbol{j}_{j+1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{G}_{u,L} & \boldsymbol{B}_{u,C} \\ \boldsymbol{A}_{i,L} & \boldsymbol{R}_{i,C} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{C,j} \\ \boldsymbol{j}_{C,j} \end{bmatrix}, \quad (2.18)$$

$$\boldsymbol{X}_{j+1}^{(k+1)} = \begin{bmatrix} \boldsymbol{i}_{u,j+1}^{(k+1)} \\ \boldsymbol{u}_{i,j+1}^{(k+1)} \end{bmatrix}; \quad \boldsymbol{H} = \begin{bmatrix} \boldsymbol{G}_{u,u} & \boldsymbol{B}_{u,i} \\ \boldsymbol{A}_{i,u} & \boldsymbol{R}_{i,i} \end{bmatrix}; \quad \boldsymbol{x}_{j+1}^{(k+1)} = \begin{bmatrix} \boldsymbol{u}_{u,j+1}^{(k+1)} \\ \boldsymbol{i}_{i,j+1}^{(k+1)} \end{bmatrix};$$

$$\boldsymbol{s}_{j+1} = \begin{bmatrix} \boldsymbol{G}_{u,e} & \boldsymbol{B}_{u,j} \\ \boldsymbol{A}_{i,e} & \boldsymbol{R}_{i,j} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{j+1} \\ \boldsymbol{j}_{j+1} \end{bmatrix}; \quad \boldsymbol{S}_{LC,j} = \begin{bmatrix} \boldsymbol{G}_{u,L} & \boldsymbol{B}_{u,C} \\ \boldsymbol{A}_{i,L} & \boldsymbol{R}_{i,C} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{L,j} \\ \boldsymbol{j}_{C,j} \end{bmatrix} \quad (2.19)$$

$$\boldsymbol{X}_{j+1}^{(k+1)} = \boldsymbol{H}\boldsymbol{x}_{j+1}^{(k+1)} + \boldsymbol{S}_{j+1} + \boldsymbol{S}_{LC,j}, \tag{2.20}$$

H represents the hybrid matrix of the circuit; $\boldsymbol{S}_{j+1}$ is the source vector corresponding to the independent voltage and current sources at the moment t_{j+1}, and $\boldsymbol{S}_{LC,j}$ represents the source vector corresponding to the companion models of the linear inductors and capacitors, at the previous time moment

$\boldsymbol{B}_{u,i}(\boldsymbol{A}_{i,u})$ are the current (voltage) gain matrix of the tree-branch (link) v.c. (c.c.) nonlinear elements in respect of the link (tree-branch) c.c. (v.c.) nonlinear elements;, $\boldsymbol{u}_{u,j+1}^{(k+1)}$ ($\boldsymbol{i}_{i,j+1}^{(k+1)}$) the voltage (current) vector of the v.c. (c.c.) tree-branch (link) nonlinear elements at the time moment t_{j+1}, and the (m+1)the iteration; $\boldsymbol{e}_{L,j}$ ($\boldsymbol{j}_{C,j}$) is the voltage (current) vector of the ideal independent voltage (current) sources from the companion schemes of the linear inductors (capacitors) at the time moment $\boldsymbol{t}_j$ (or at the previous time steps).

The nonlinear resistor characteristics approximated by piecewise linear continuous curves have, for the time moment t_{j+1}, and the (k+1)the iteration:

for the v.c. nonlinear resistors:

$$i_{Ru,j+1}^{(k+1)} = \hat{i}_{Ru}(u_{Ru,j+1}^{(k+1)}) = G_{du}\left(s_{j+1}^{(k)}\right)\cdot u_{Ru,j+1}^{(k+1)} + j_{Ru}\left(s_{j+1}^{(k)}\right),$$

$$u_{Ru}^{-}\left(s_{j+1}^{(k)}\right) \le u_{Ru,j+1}^{(k+1)} \le u_{Ru}^{+}\left(s_{j+1}^{(k)}\right) \tag{2.21}$$

for the c.c. nonlinear resistors

$$u_{Ri,j+1}^{(k+1)} = \hat{u}_{Ri}(i_{Ri,j+1}^{(k+1)}) = R_{di}\left(s_{j+1}^{(k)}\right)\cdot i_{Ri,j+1}^{(k+1)} + e_{Ri}\left(s_{j+1}^{(k)}\right),$$

$$i_{Ri}^{-}\left(s_{j+1}^{(k)}\right) \le i_{Ri,j+1}^{(k+1)} \le i_{Ri}^{+}\left(s_{j+1}^{(k)}\right) \tag{2.22}$$

The current expression of a v.c. nonlinear capacitor, when its characteristic is approximated by piecewise linear continuous curve, for the time moment t_{j+1}, and the (k+1)th iteration (using the backward Euler integration algorithm):

$$i_{Cu,j+1}^{(k+1)} = G_{dCu}\left(s_{j+1}^{(k)}\right)\cdot u_{Cu,j+1}^{(k+1)} + \hat{j}_{Cu,j+1} - j_{Cu,j}, \tag{2.24}$$

unde: $G_{dCu}\left(s_{j+1}^{(k)}\right) = \dfrac{C_{du}\left(s_{j+1}^{(k)}\right)}{h}$; $\hat{j}_{Cu,j+1} = \dfrac{Q_{Cu}\left(s_{j+1}^{(k)}\right)}{h}$; $j_{Cu,j} = \dfrac{q_{Cu,j}}{h}$; $j_{Cu,j+1,j} = \hat{j}_{Cu,j+1} - j_{Cu,j}$.

The expression of a c.c. nonlinear coil, when its characteristic is approximated by piecewise linear continuous curve, for the time moment t_{j+1}, and the (k+1)th iteration (using the backward Euler integration algorithm):

$$\varphi_{Li,j+1}^{(k+1)} = \hat{\varphi}_{Li}\left(i_{Lu,j+1}^{(k+1)}\right) = L_{di}\left(s_{j+1}^{(k)}\right)\cdot i_{Lu,j+1}^{(k+1)} + \Phi_{Li}\left(s_{j+1}^{(k)}\right).$$

$$i_{Li}^{-}\left(s_{j+1}^{(k)}\right) \le i_{Li,j+1}^{(k+1)} \le i_{Li}^{+}\left(s_{j+1}^{(k)}\right).$$

(2.25)

where $s_{j+1}^{(k)}$ is a certain segment at the moment t_{j+1} and iteration (k), and parameters $L_{di}\left(s_{j+1}^{(k)}\right)$ and $\Phi_{Li}\left(s_{j+1}^{(k)}\right)$ represents the slope, the differential inductance, and the ordinate of origin of the corresponding segment $s_{j+1}^{(k)}$. The range $\left[i_{Li}^{-}\left(s_{j+1}^{(k)}\right), i_{Lci}^{+}\left(s_{j+1}^{(k)}\right)\right]$ represents the definition domain of the segment $s_{j+1}^{(k)}$.

Substituting the linear piecewise characteristics of the nonlinear circuit elements into the equations (1), we obtain:

$$\begin{bmatrix} \left[\boldsymbol{G}_{du}\left(s_{j+1}^{(k)}\right)-\boldsymbol{G}_{u,u}\right] & -\boldsymbol{B}_{u,i} \\ -\boldsymbol{A}_{i,u} & \left[\boldsymbol{R}_{di}\left(s_{j+1}^{(k)}\right)-\boldsymbol{R}_{i,i}\right] \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_{u,j+1}^{(k+1)} \\ \boldsymbol{i}_{i,j+1}^{(k+1)} \end{bmatrix} = \begin{bmatrix} \boldsymbol{G}_{u,e} & \boldsymbol{B}_{u,j} \\ \boldsymbol{A}_{i,e} & \boldsymbol{R}_{i,j} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{j+1} \\ \boldsymbol{j}_{j+1} \end{bmatrix} + \\ + \begin{bmatrix} \boldsymbol{G}_{u,L} & \boldsymbol{B}_{u,C} \\ \boldsymbol{A}_{i,L} & \boldsymbol{R}_{i,C} \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_{C,j} \\ \boldsymbol{j}_{C,j} \end{bmatrix} - \begin{bmatrix} \hat{\boldsymbol{j}}_{Cu,j+1}-\boldsymbol{j}_{Cu,j} \\ \hat{\boldsymbol{e}}_{Li,j+1}-\boldsymbol{e}_{Li,j} \end{bmatrix} - \begin{bmatrix} \boldsymbol{j}_{u}\left(s_{j+1}^{(k)}\right) \\ \boldsymbol{e}_{i}\left(s_{j+1}^{(k)}\right) \end{bmatrix} \tag{2.29}$$

The formulation algorithm for hybrid equations (1) for nonlinear analog circuits involves the following steps:

P1. Select a normal tree under the conditions mentioned above;

P2. First we replace all the nonlinear elements in voltage controlled (c.u) with equivalent ideal independent voltage sources, and the ones in current controlled (c.i.) with equivalent ideal independent current sources;

P3. Second we replace linear coils (magnetic coupled or not) and linear capacitors with equivalent resistive companion discrete circuits, (at appropriate time = + h, h being the integration step), associated with a default integration algorithm. Thereby, we obtain a linear resistive circuit which contain linear resistors (including the ones from the equivalent companion circuits of linear coils and capacitors), independent voltage and current sources (including those which have been substituted the nonlinear elements of circuit, c.u respectively c.i.) and controlled sources;

P4. It examines with a suitable simulation program the resistive circuit obtained in step P3. Thus obtaining the full symbolic expressions or numeric-symbolic of currents vector of nonlinear elements c.u, and the tensions of nonlinear elements c.i depending on: vector electromotive voltage (currents) of the ideal independent voltage source (current), depending on the vector electromotive tension (current) of equivalent companion schemes of linear coil (linear capacitors) and depending on the independent variables. From the expression vectors iu and ui is determined the hybrid matrix H, vector Vj+1 source vector at the time determined by independent voltage and current sources and the vector VLC - source vector corresponding to independent sources from discrete and resistive models, associated with a default integration algorithm of linear coils and capacitors;

The algorithm of formulation and solving hybrid equations can be used for a large class of nonlinear analog circuits.

The goal is to obtain a simple, flexible and general algorithm.

In order to calculate the symbolic expressions of the hybrid matrices we use the simulator Maple 15. Maple (developed by Maplesoft) is a math software that is used to analyze, explore, visualize, and solve mathematical problems. Maple has a powerful math engine and covers also aspects of technical computing, including visualization, data analysis, matrix computation, and connectivity. This makes it the ideal tool for both education and research.

Some of the helpful functions in Maple :coeff - extract a coefficient of a polynomial, collect - collection after power coefficients, subs - substitute in an expression subexpression, read File- read a file and executes it.

At step 4, the suitable simulation program that examine the resistive circuit is Asinom.

We use the program Asinom to calculate the solution of the entire circuit

(tension and current of all analyzed circuit). The two output files contains one the modified nodal-analysis equations and the other output file has the tensions and currents of all analyzed circuit.

Chapter 3

Calculation of hybrid equations on analog circuits

Consider the nonlinear circuit in Figure 3.1 that operates in variable mode. It contains a c.i. nonlinear resistor, a c.u. nonlinear capacitor, a voltage controlled current source, a voltage independent source, a linear resistor, a linear capacitor and a linear coil.

The nonlinear circuit elements are replaced with, but with ideal independent sources of voltage and current respectively, and the dynamic linear circuit elements (capacitor C3 and coil L6) are replaced with discrete resistive models, associated with an implicit algorithm of numerical integration (Euler default), resulting in the equivalent scheme in figure 3.2.

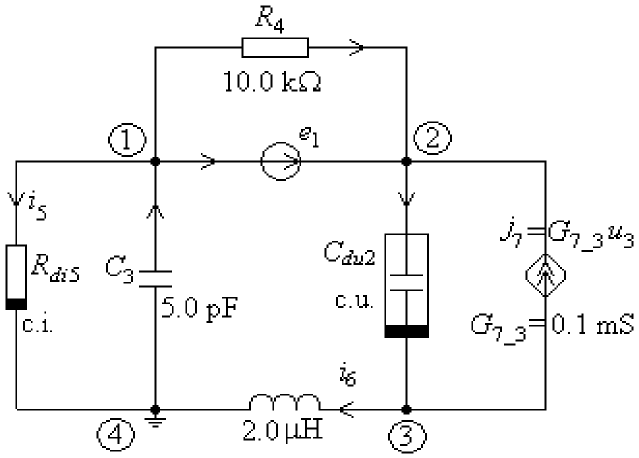


Fig. 1. Initial scheme of the circuit

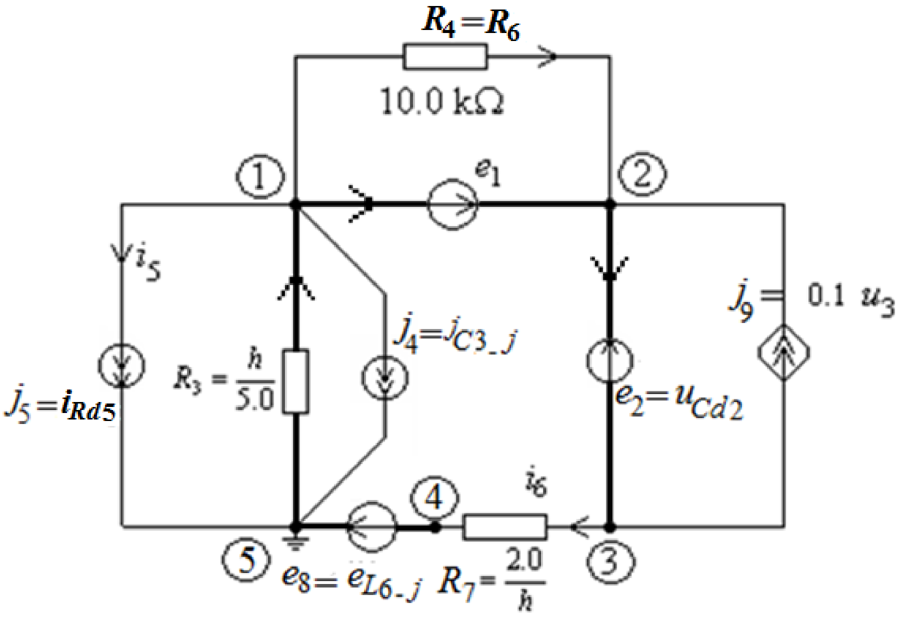


Fig. 2 equivalent circuit diagram of the Figure 1, where nonlinear resistors c.i or c.u. are replaced with the ideal source equivalent current independent

Here are the Kirchhoff's equations for figure 2:

+ (+1/((h/C3))+1/(R4))*V1 + (-1/(R4))*V2 + (+1)*I1 = -jC3_j-iLd5

+ (-1/(R4)+G9_3)*V1 + (+1/(R4))*V2 + (-1)*I1 + (-1)*I2 = 0

+ (-G9_3)*V1 + (+1/((L6/h)))*V3 + (-1/((L6/h)))*V4 + (+1)*I2 = 0

+ (-1/((L6/h)))*V3 + (+1/((L6/h)))*V4 + (+1)*I8 = 0

+ (+1)*V1 + (-1)*V2 = -E1

+ (-1)*V2 + (+1)*V3 = -uCd2

+ (+1)*V4 = -eL6_j

In order to calculate the symbolic expressions of the hybrid matrices we use the simulator Maple 15. The program developed below in Maple has a simple, flexible and general algorithm, see Appendix A.

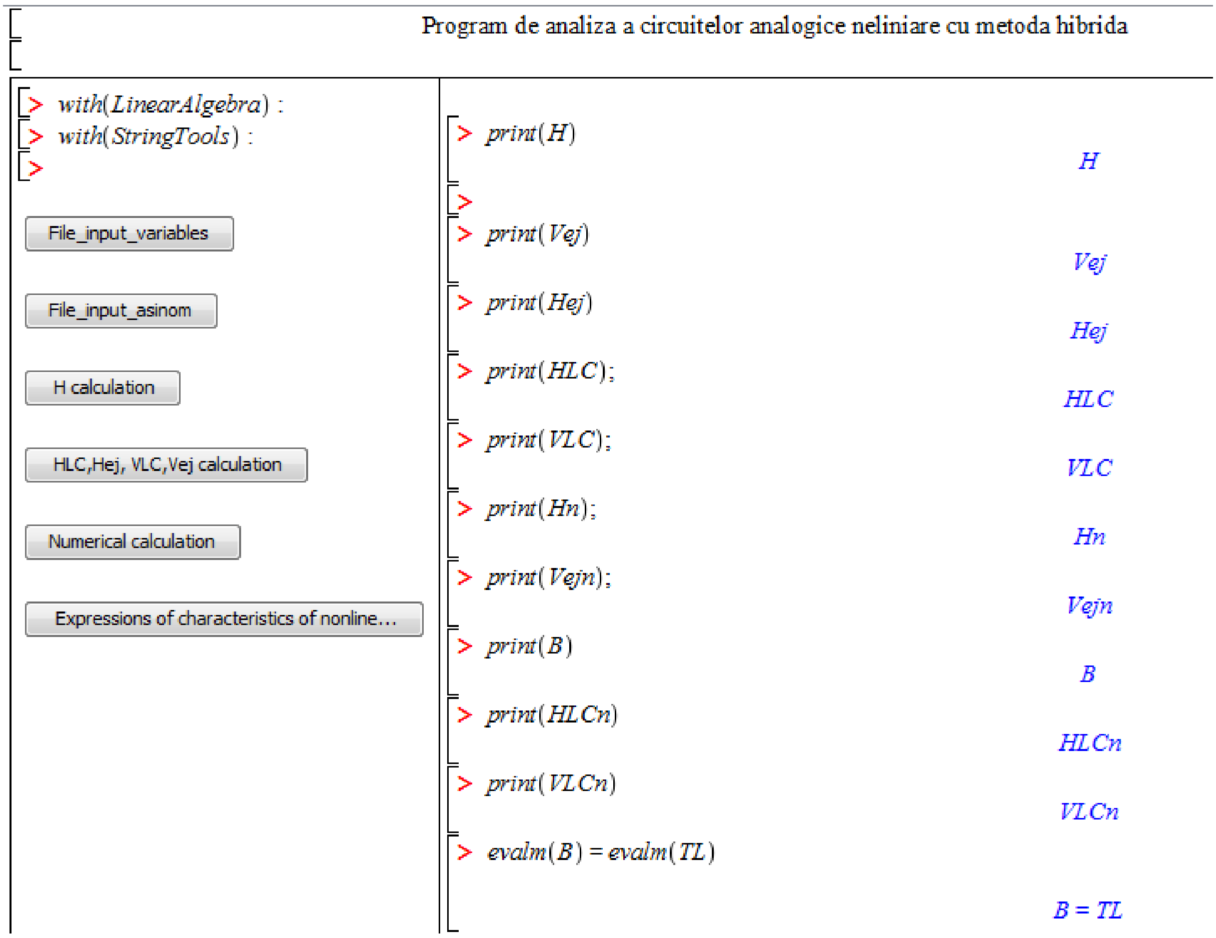


Fig 3.3 Interface of the program of automatic formulation of hybrid equations

The analysis in the time domain of the circuit can be performed with a calculation program in Matlab, using the hybrid equations generated above, or with the ACA program - starting from the description of the circuit through a netlist input file. The characteristics of non-linear circuit elements are specified by dots.

Input file *ex1_Hib.nln*, is:

7	epsilon=0.00001	10.0 50.0
4	puncte Cdu2:	80.0 60.0
1 2 e1 e=e1(i)		100.0 60.5
2 3 Cdu2 pct=9	-100.0 -60.5	
4 1 C3 c=5.0	-80.0 -60.0	puncte Rdi5:
1 2 R4 r=10.0	-10.0 -50.0	
1 4 Rdi5 pct=3	-1.0 -10.0	-5.5 -200.0
3 4 L6 l=4.0	0.0 0.0	0.0 0.0

3 2 j7(u) 3 gt=0.1 1.0 10.0 10.5 200.0

In the input file ex1_Hib.nln the characteristic parameters and sizes of the circuit are expressed as follows: resistances in kΩ, capacities in pF, inductances in μH, time in ns, voltages in V, currents in mA, frequency GHz and electrical charges in nC. The ACA program with Matlab allows Fourier analysis (frequency domain analysis), calling the fft.m. subroutine.

Considering $e_1 = 40.0*\sin(2\pi(f_1 - f2)*t+1/57)$ (f_1 = 1GHz, f_2 = 0.1 GHz), and calling the above program we obtain the results presented in Table 3.1.

The ACA - Analog Circuit Analysis program is developed in C ++ language, using the Microsoft Visual Studio application. The ACA program uses the modified nodal method and is based on the numerical integration of ordinary differential equations, the generalized one-order regressive method (Euler algorithm by default), [42-43].

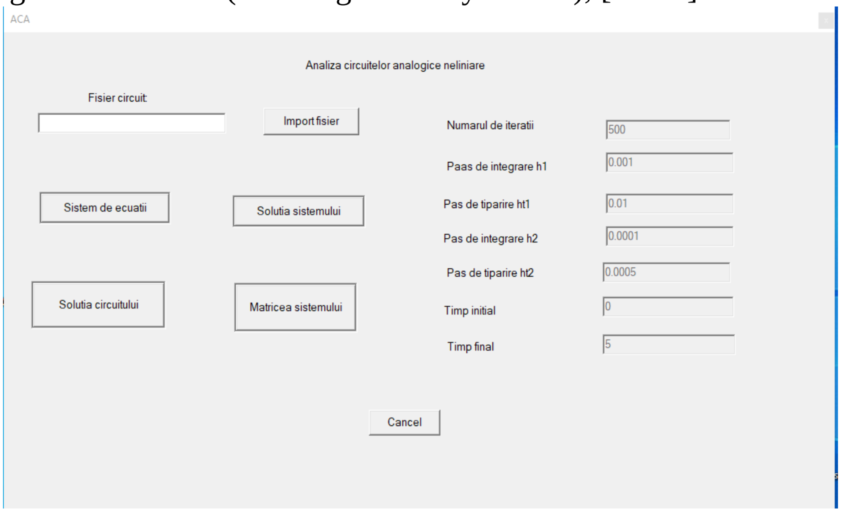
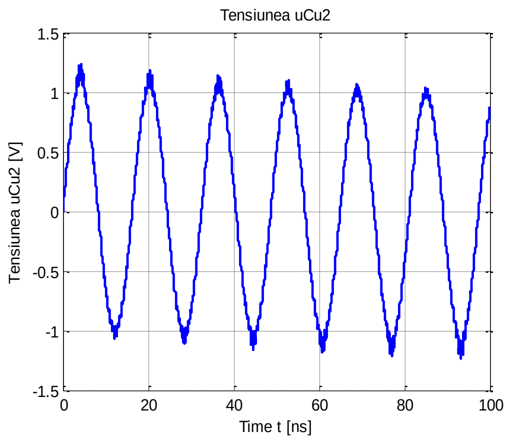
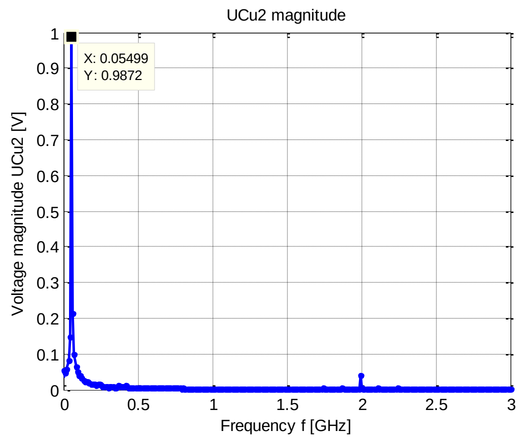
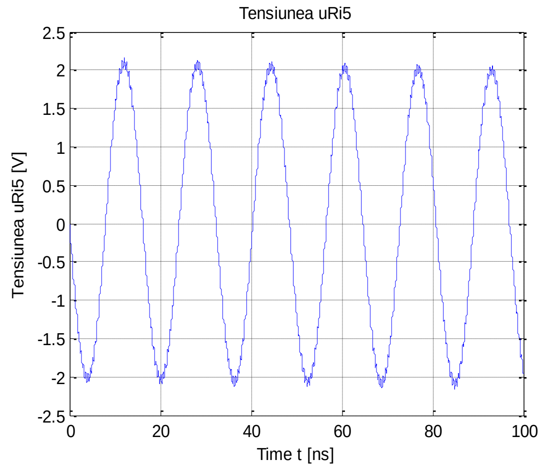


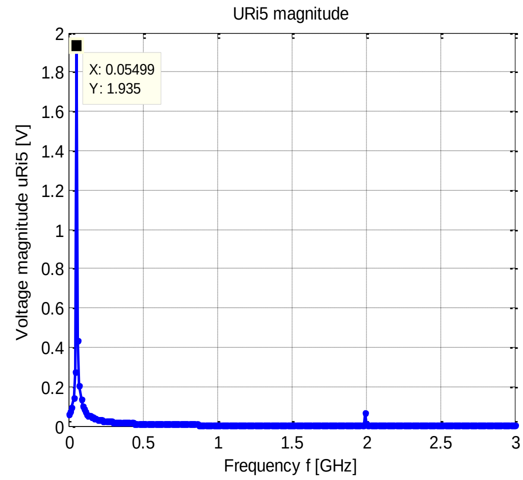
Fig. 3.8. Graphical interface of the ACA program.

Tabelul 3.1

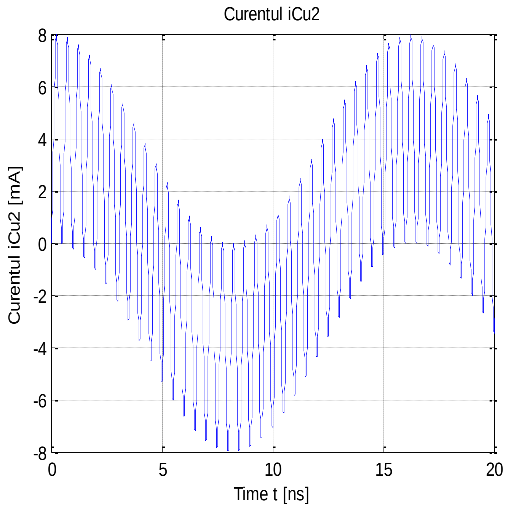
Time domain analysis	**Frequency analysis**
 Time variation of voltages u_{Cu2}.	 Voltage frequency variation u_{Cu2}.



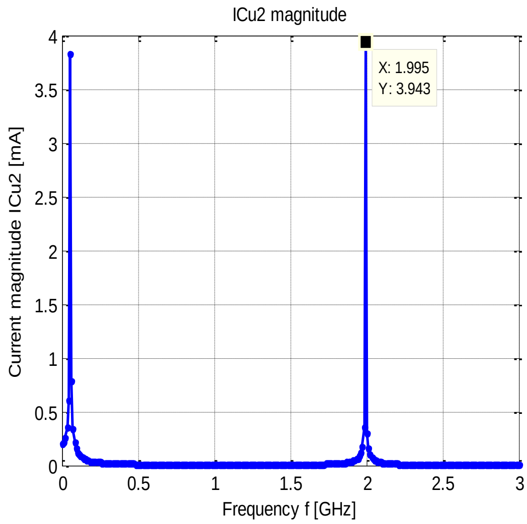
Time variation of voltages u_{Ri5}.



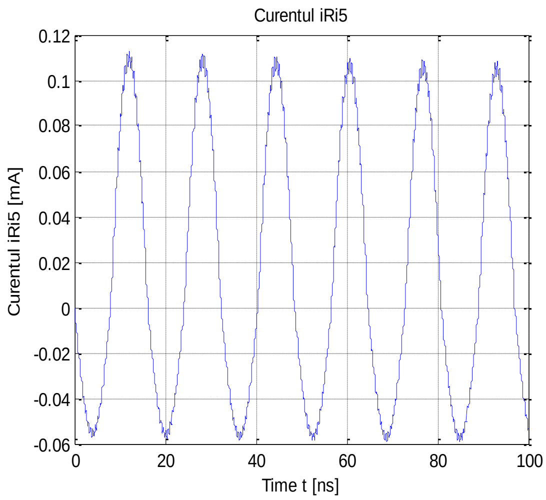
Voltage frequency variation u_{Ri5}.



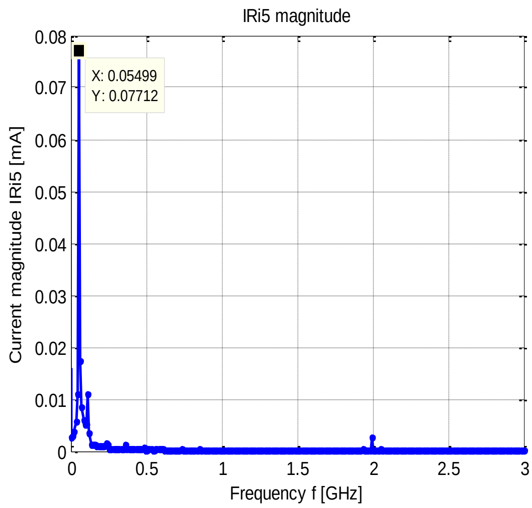
Time variation of current i_{Cu2}.



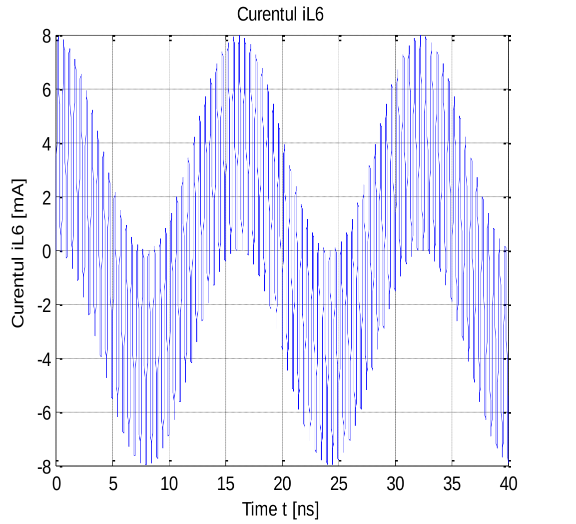
Current frequency variation i_{Cu2}.



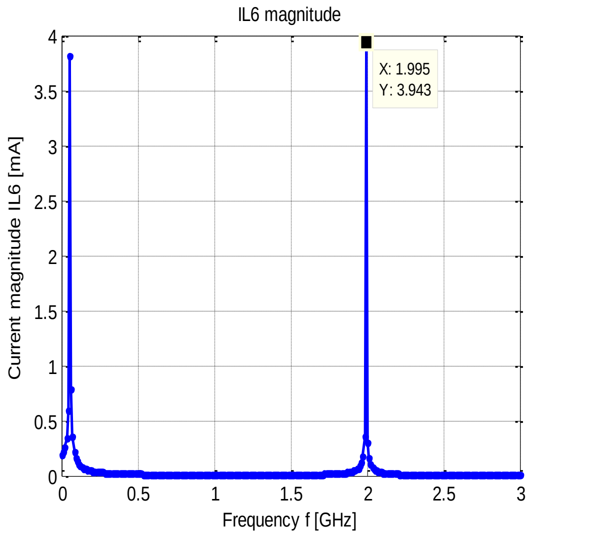
Time variation of current i_{Ri5}.



Current frequency variation i_{Ri5}.



Time variation of current i_{L6}.



Current frequency variation i_{L6}.

Chapter 4

Applicability of the hybrid matrix in practice

In order to achieve an earthing system in accordance with European standards, it is mandatory to study the electrical behavior of the soil. Soil resistivity influences the dispersion resistance of the earth connection. Knowing the resistivity of the soil in the place where you want to make the earthing is the responsibility of a design engineer and he must take into account that the resistivity of the soil varies depending on temperature, moisture content and depth at which the electrodes that form the earthing are buried. The electrical properties of the soil are characterized by its resistivity. It is difficult to determine its value due to the fact that the soil does not have a homogeneous structure, being formed by layers of different materials.

Figure 3.6 shows the equivalent nonlinear resistive analog circuit where the ground resistivity is $\rho = 200$ [$\Omega \cdot$ m]. This practical application deals with solving the nonlinear resistive analog circuit in figure 3.6. This nonlinearity is due to the values of specific soil strengths. According to I7 / 2011, (according to NTE 007/08/00) the values of specific soil resistances vary depending on the geographical location of the globe, as follows [48]:
- sand with 10% humidity 1Km / W;
- clay soil 0.65 Km / W;
- sandy soil with humus or clay 0.65 Km / W;
- dry sand (0% humidity) 3.00 Km / W;
- ordinary dry land 3.00 Km / W

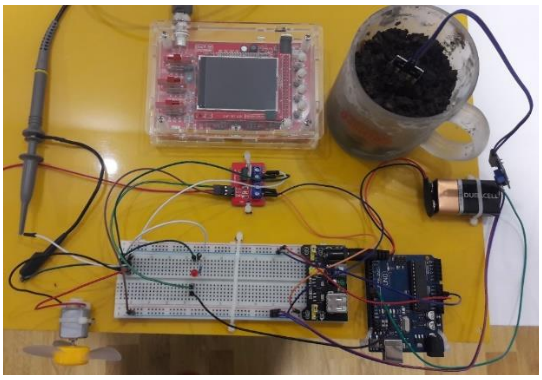


Fig 4.4 Practical application layout with assembled components

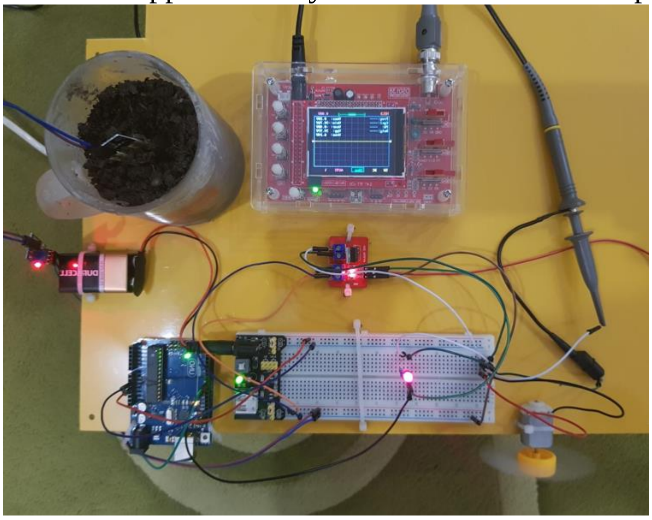


Fig. 4.5. Scenario when the engine starts due to dry soil (low humidity)

We will take as a standard the value of the specific resistance of the soil 1Km / W (kelvin meter / watt) for sandy soil with 10% humidity.
In the soil sample (the one in the cup) depending on the humidity, the temperature, the depth at which the sensor is found, we have several values of the specific resistances.
If the measured value is lower than the standard value then it means that the ground is wet and the water pump represented by the 5 V DC motor will not start.
If the measured value is higher than the standard value then it means that the ground is dry / arid and the water pump represented by the 5 V DC motor will start until the value of the specific resistance of the ground reaches the value of the standard resistance.

The practical application consists in modeling the variation of the specific resistances of the soil.
The components of the practical application used consist of:
• IRF520 power transistor mode
• Arduino board
• module with humidity sensor
• DSO138 oscilloscope kit

Chapter 5

Final conclusions, contributions, future research directions

The following are the main contributions originally made by the author in this PhD thesis:

- The paper begins with a carefully selected and up-to-date documentation on the analysis, modeling, simulation and design of linear and / or nonlinear analog circuits. The main problems that occur in the simulation of nonlinear analog circuits are presented. The methods of analysis, modeling, simulation and design of existing linear and / or nonlinear analog circuits and the most used calculation programs are identified. The results are synthesized in a useful manner and thus constitute a useful tool for the development of further research in the field;
- The original implementation of a generalized method of analysis of nonlinear circuits called hybrid analysis. It is called hybrid because the mathematical model of the analyzed circuit has as independent variables both currents and voltages;
- The advantages of the hybrid method are highlighted in relation to the other existing methods of analysis and it is, in particular, compared to the modified nodal method. The hybrid method has equations consisting of a relatively small number of variables which reduces the computation time and memory used for computer analysis of nonlinear circuits. Another advantage is the separation of the linear part from the non-linear part of the circuit. The equations that describe the operation of the linear part are formulated only once, at the beginning of the calculation process, which facilitates the calculation effort. These equations remain unchanged during the iterative solving process. In this way the calculation effort is significantly optimized;
- In the first chapter are presented briefly some fundamental notions about the analysis of nonlinear and parametric circuits exactly in order to understand the algorithm of the hybrid analysis method;
- Chapter 2 presents the hybrid analysis method for resistive nonlinear circuits as well as for those that contain dynamic circuit elements (capacitors and coils). Linear and / or nonlinear coils, linear and / or nonlinear capacitors and nonlinear resistors are replaced by equivalent discrete, resistive circuits and associated with an implicit numerical integration algorithm. Once formulated, hybrid circuit equations can be solved by a variety of iterative techniques, the most general and most widely used being the method based on the Newton-Raphson algorithm. Analysis of nonlinear circuits in dynamic mode uses Kirchhoff's theorems, companion diagrams associated with an implicit algorithm of numerical integration (default Euler), the characteristic equations of linear resistors and the definition relations of the ordered sources. In general, the hybrid method manages to find a solution for the vast majority of the analyzed circuits. Obtaining a solution cannot be guaranteed even if the circuit and its specifications are correct, due to the nonlinearity of the equations describing the circuit and the imperfection of the analytical models of the devices. In most cases when a solution cannot be obtained, the failure is due to the problems given by the circuit, either due to the way it is described, or due to the fact that the analyzed circuit is not functional;
- The more advanced the complexity of the circuit, the greater both the difficulty of formulating criteria for the existence of solutions and the difficulty of applying them. In such cases, the application of criteria with a more advanced degree of generality is avoided and probing is used to find the solution (s), possibly by several methods. The criteria specific to resistive circuits are valid for both stationary and dynamic regimes if the analysis is based on equivalent resistive companion schemes;
- In chapter 3 are presented several examples of nonlinear electrical circuits. These are analyzed with the help of two circuit analysis programs, namely: Asynomus - Symbolic Analysis based on the Modified Nodal Method and ACA - Analog Circuit Analysis Program. Both examples presented are solved by the modified nodal method and by the hybrid method in order to be able to finally make a comparison between the two methods. Analyzing the graphical results obtained, it is observed that the variations of currents and voltages obtained by the modified nodal method and the

hybrid method are similar. The only differences that occur are due to the integration step (h). The smaller the integration step, the smaller the differences;

- For all the examples presented in the paper are described, step by step, their solving algorithms;
- Chapter 4 deals with solving a nonlinear resistive analog circuit with a practical application. This nonlinearity is due to the values of specific soil strengths. With a (humidity) sensor we will measure the humidity value in the discretized soil. If the soil is dry sandy (0% humidity) the digital output of the humidity sensor will be set to the logical “1”. This signal is connected to digital port 8 of the Arduino Uno development board, a port set as the input port. The digital port 9 of the Arduino Uno development board is set as the Output port. This signal represents the control of the Mosfet transistor located on the power transistor module IRF520 that drives a pump (simulated in the practical application with the help of a 5 V DC motor).

This thesis presents new approaches of analyzing the nonlinear resistive circuit analysis. The hybrid analysis method of the nonlinear analog circuits presents the advantage of separation between the linear and the nonlinear part. This allows the computation only once at the beginning of the iteration process, of those parts of the circuit equations that exclusively depends on the parameters of the linear terms. A significant efficiency in circuit analysis and an improvement of the accuracy in the numerical calculations are obtained by combining this hybrid procedure with a very efficient implicit integration algorithm, in which only the symbols of the parameters corresponding to the nonlinear circuit elements are considered. Also the dynamic elements are replaced by discrete resistive models associated with an implicit numerical integration algorithm.

Another advantage of the hybrid-method in comparison with nodal-analysis method is flexibility by allowing the nonlinear elements to be either voltage-controlled (c.v.) or current-controlled (c.c.).

Based on the results obtained in this thesis, the following main directions for further research can be identified:

 Adapting the software presented in the thesis to the current and future needs of engineering science regarding the analysis of analog and digital circuits;

 The use of harmonic balance and Volterra series for the study of nonlinear circuits in non-sinusoidal periodic regime ;.

 Introduction of new tools (calculation tools) by circuit designers that allow them to understand and exploit the nonlinearity of circuits for the most useful processing;

 Studying the nonlinear behavior of the circuit in a graphical way, facilitating the development of a qualitative assessment for nonlinear analog circuits.

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Appendix Program code developed in MAPLE for the analysis of nonlinear circuits with the hybrid method

Open file and read data

```
>openDialogue := module()
  local MAPLEt, ModuleApply;
  global fd;uses MAPLEts:-Elements;
  MAPLEt := MAPLEt(FileDialog['FD1'](
    ':-filefilter' = "txt",
    ':-filterdescription' = "Text Files",
    ':-onapprove' = Shutdown( ['FD1'] ),
    ':-oncancel' = Shutdown() ));
  ModuleApply := proc()
    local G;
    G := MAPLEts:-Display(MAPLEt);
   if G::list and numelems(G)>0 and G[1]<>"" then
    fd:=fopen( G[1],READ,TEXT)
    end if;
    return NULL;
  end proc;
end module:
global first,second,third,four;
OpenDialogue();

>first := readline(fd);
>second := readline(fd);
>third := readline(fd);
>four := readline(fd);
>fclose(fd);


>d := Split(third);
>for j to nops(d) do parse(d[j], statement) end do;
```

```
rez1 := simplify(rez1)
>covar := Split(first);
>var := Split(second);
>print(nops(var));
>for j to nops(var) do varp[j] := parse(var[j]) end do;
>for j to nops(covar) do covarp[j] := parse(covar[j]) end do;
```

Vector calculation of covariates

```
> for j to nops(covar) do vectorCovar[j] := collect(simplify(subs(rez1, covarp[j])),
[seq(varp[i], i = 1 .. nops(var))]) end do;
```

Calculation of hybrid matrix H

```
> for j to nops(covar) do for i to nops(covar) do H[j][i] := 0 end do end do;
> for j to nops(covar) do for i to nops(covar) do H[j][i] := coeff(vectorCovar[j], varp[i]);
print(H[j][i]) end do end do;
```

Calculation of matrix HLC and Vej

```
dinamic := Split(five);
for j from 1 to nops(dinamic) do
din[j] := parse(dinamic[j]) end do;
if din[1] <> 0 then print*ana else print*blabla end if;

if din[1]<>0 then
print (ana);
for j from 1 to nops(covar) do
for i from 1 to nops(dinamic) do
HLC[j][i] := coeff(vectorCovar[j], din[i]);
end do;
end do;
end if;
ss := Split(six);

for j from 1 to nops(ss) do surse[j] := parse(ss[j]) end do;
for j from 1 to nops(covar) do
```

```
for i from 1 to nops(ss) do Hej[j][i] := coeff(vectorCovar[j], surse[i]); print(Hej[j][i]) end do
end do;
M3 := Matrix(nops(covar), nops(ss), 0);
for j from 1 to nops(covar) do
for i from 1 to nops(ss) do
M3[j, i] := Hej[j][i] end do end do;
print(M3);
M4 := Matrix(nops(ss), 1, 0);
for i from 1 to nops(ss) do M4[i] := surse[i] end do;
Vej := Multiply(M3, M4);
if din[1] <> 0 then HLCmatrix := Matrix(nops(covar), nops(dinamic), 0);
for j from 1 to nops(covar) do
for i from 1 to nops(dinamic) do
HLCmatrix[j, i] := HLC[j][i] end do end do;
dinmatrix := Matrix(nops(dinamic), 1, 0);
for j from 1 to nops(dinamic) do
dinmatrix[j] := din[j] end do;
VLC := Multiply(HLCmatrix, dinmatrix) end if;
end use;
```

Numeric calculation of above matrix

```
num := Split(seven);
for j from 1 to nops(num) do valori[j] := parse(num[j]) end do;
if din[1] <> 0 then VLCn := subs(valori[1], evalm(VLC));
print(SLCn); HLCn := subs(valori[1], evalm(HLCmatrix)) end if;

Hejn := subs(valori[1], evalm(M3));
Vejn := subs(valori[1], evalm(Vej));
Hmatrix := Matrix(nops(covar), nops(covar), 0);

for j from 1 to nops(covar) do for i from 1 to nops(covar) do
 Hmatrix[j, i] := H[j][i]; print(Hmatrix[j][i]) end do end do;
Hn := subs(valori[1], evalm(Hmatrix));
end use;
```

Expressions of the characteristics of nonlinear circuit elements, linearized in portions, corresponding to any combination of segments and hybrid equations:

```
segn := Split(eight);
for j from 1 to nops(segn) do segmente[j] := parse(segn[j]); end do;
segmentematrix := Vector(nops(segn), 0);
for j from 1 to nops(segn) do segmentematrix[j] := segmente[j]; end do;
C := Matrix(nops(covar), nops(covar), 0);
for j from 1 to nops(covar) do
for i from 1 to nops(covar) do
if i = j then C[j, i] := segmentematrix[i]-Hn[j, i];
else C[j, i] := -Hn[j, i]; end if; end do; end do;
ninevector := Split(nine);
for j from 1to nops(ninevector) do suiv[j] := parse(ninevector[j]) end do;
Sui := Vector(nops(ninevector), 0);
for j from 1 to nops(ninevector) do Sui[j] := suiv[j]; end do;
if din[1] <> 0 then TL := evalm(Vejn+VLCn-Sui); else TL := evalm(Vejn-Sui); end if;
fourvector := Split(four);
for j from 1 to nops(fourvector) do xv[j] := parse(fourvector[j]); end do;
x := Vector(nops(fourvector), 0);
for j from 1 to nops(fourvector) do x[j] := xv[j]; end do;

B := Multiply(C, x);
evalm(B) = evalm(TL)
end use;
```