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The study of critical cases for stability of solutions of switched systems of delay differential equations with applications in engineering

PhD Thesis Summary

Keywords: *switched systems; state and control time-delay; Lyapunov-Krasovskii functional; critical case of stability; Malkin approach; predictive feedback method; electrohydraulic servomechanism; equilibrium asymptotic stability; numerical simulation*

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Introduction

This PhD Thesis was a challenge both from a mathematical point of view and from the perspective of applied engineering.

From a mathematical point of view, the Thesis is a study of equilibrium stability in the sense of Lyapunov for some systems described by nonlinear differential equations with delayed argument having a switching structure based on uncontrolled switching rules. Thus, the approached problem becomes a complex one, with a certain character of uniqueness in the literature. For some of these systems, the case of critical equilibrium stability is analysed using the Lyapunov-Malkin paradigm. Two mathematical models are proposed, both based on the use of some Lyapunov-Krasovskii functionals. In one model, the problems raised by the nonlinearity of the system: the switching character, the introduction of a state delay, and the consideration of a critical case in which one or more eigenvalues of the system are null, are studied simultaneously. In the second mathematical model, the case of a nonlinear, switched system, with delay on control, is studied. A solution based on the predictive control method is given.

In the literature, there are quite a few papers on the analysis of delayed linear systems using Lyapunov-Krasovskii functionals. Delayed nonlinear systems have long time been avoided or treated with less interest in engineering studies, maybe due to the high degree of difficulty of such a framework. However, recently, the attention of specialists in the field is beginning to turn to this type of systems. Concerning switched systems, there is also a growing interest here lately, especially due to the frequent applications in the field of technology and communications.

The novelty appears also when the two themes, of the nonlinear systems with delay and of the systems with switching, are looked at simultaneously. This direction is the subject of this PhD Thesis, the topic being widely developed in the following chapters.

From an engineering point of view, it is proposed an application to hydraulic servomechanisms, especially in the case of the electrohydraulic servomechanism, characterized by structural switching with an internal and uncontrolled switching rule.

The Thesis is structured in four chapters being accompanied by Introduction, List of Publications, and Original results preceding the first Chapter, and by Conclusions and References after the last Chapter.

Therefore, in summary, in Chapter 1, an overview of the delay differential equations is given. Necessary definitions are stated and the classical definitions of stability in the Lyapunov sense are also given. At the same time, the notion of switched system and the ways they are handled are introduced. What is interesting about switched systems is that even if the constituent subsystems are stable, the system as a whole may be not guaranteed stable. Moreover, the switching rule can stabilize unstable subsystems, or it can destabilize stable subsystems. In the case of the Thesis, the difficulty of the analysis also derives from the fact that the switching law is a structural, uncontrolled one.

In Chapter 2, the problem of state delay simultaneously with the issue of the critical case of equilibrium stability is addressed. This leads to the application of a Malkin-type mathematical apparatus, combined with the use of complete Lyapunov-Krasovskii functionals. Following some transformations of variables, the initial system consisting of five nonlinear differential equations is rewritten in the form specific to the Lyapunov-Malkin approach which consists of some fourth order system and an equation that contains only nonlinear terms. Thus, the stability of the nonlinear system with one eigenvalue equal to zero is settled based on some stability conditions on the linear part of the fourth order system brought to the Lyapunov-Malkin form. A theorem for the critical case of equilibrium stability for the uncontrolled switched nonlinear system with state delay is proved.

Another type of nonlinear systems with delay and structural switching is studied in Chapter 3. This time, the delay is considered on the control variable and the critical case of equilibrium stability is eluded by taking into account a mathematical model more appropriate of the real world object. A predictive feedback control method is proposed. In this framework, a theorem for equilibrium stability for a dynamical nonlinear system with delay on control and uncontrolled, structural switching is given.

Chapter 4 is dedicated to the applications of the theory developed in Chapter 2 and 3 to hydraulic servomechanisms, specifically to those used in aviation. In Section 4.1, the general description of the hydraulic servomechanism is given. Moreover, some mathematical models with switching and delay are proposed. Section 4.2 applies the theory from Chapter 2, and Section 4.3 is the application of the theory from Chapter 3. The conservative character given by the use of the Lyapunov-Krasovksii functionals is highlighted by numerical applications. In fact, it is identified, both analytically and numerically, the maximum value of the delay for which the system remains stable, this being relevant on the related graphs. Section 4.3 is dedicated to a study in the discrete field within the method of predictive control and the mode in which a control law is synthesized when the system has delay on control.

All the results obtained during the scientific research within this PhD Thesis have been published in indexed journals as Web of Science or international databases and presented at national and international scientific conferences and seminars.

1. Theoretical background. Delay differential equations and switched systems

The topic of delayed differential equations (DDE) appeared as a natural need to describe as accurately as possible phenomena that occur in everyday life and whose current action is influenced by the history of their evolution. For some phenomena, processes or physical systems, a mathematical model is more appropriate to reality as it considers the influence from a recent time interval. As expected, this leads to a significant increase in the complexity of the issue addressed.

A notable difference between a system with and without time delay is that a delay introduced in a differential equation produces an infinite dimensional system. The characteristic equation of a DDE is a transcendental equation and not an algebraic equation, as in the case of linear differential equations without delay.

Consider the following nonlinear differential system with constant delay

$$\dot{x}(t)=f\left(x(t),x(t-h)\right),\quad t\geq t_0,\quad h=\text{const}>0 \tag{1}$$

with $f:D\times D\to\mathbb{R}^n$, $D\subset\mathbb{R}^n$, a locally Lipschitz and continuous vector-valued function. Also, consider an initial time $t_0\geq 0$ and an initial function $\varphi\in C\left([-h,0],\mathbb{R}^n\right)$ defined on the normed space of continuous vector functions with the standard uniform norm $\|\varphi\|_h=\sup_{-h\leq\theta\leq 0}\|\varphi(\theta)\|$ (Kharitonov, 2013). The DDE system considered in the present Thesis is *nonlinear, deterministic, autonomous, with single constant delay*.

The state $x_t(t_0,\varphi)(\theta):=x(t+\theta;t_0,\varphi),\theta\in[-h,0]$ at a time instant $t\geq t_0$ along a solution $x(t;t_0,\varphi)$ is defined as the restriction of this solution on $[t-h,t]$. Usually, the arguments t_0 and φ can be omitted given us the possibility to write $x(t)$ instead of $x(t;t_0,\varphi)$ and x_t instead of $x_t(t_0,\varphi)$. With these notations, system (1) is a particular case of the general form

$$\dot{x}(t)=f\left(x_t\right),\ x_t\in C\left([-h,0],\mathbb{R}^n\right) \tag{2}$$

with f a functional defined on $C\left([-h,0],\mathbb{R}^n\right)$.

The initial value problem (the Cauchy problem) consists in finding a solution $x(t;t_0,\varphi)$, of the system (1) that satisfies the initial condition

$$x(\theta;t_0,\varphi):=x(t_0+\theta)=\varphi(\theta),\theta\in[-h,0]. \tag{3}$$

A common method to solve the Cauchy Problem for DDE is the method of steps or the method of successive integrations (Kolmanovskii & Myshkis, 1999), (Kálmar-Nagy, 2009).

Theorem 1. (existence and uniqueness) ((Kharitonov, 2013) §1.2. Th. 1.1). *Consider the time-delay system (2) with the functional* $f : C\left(\left[-h, 0\right], \mathbb{R}^n\right) \to \mathbb{R}^n$ *satisfying the following conditions:*

a) for any $\alpha > 0$ *there exists* $M(\alpha) > 0$ *such that* $\| f(\varphi)\| \le M(\alpha),\ \varphi \in C\left(\left[-h, 0\right], \mathbb{R}^n\right)$ *and* $\|\varphi\|_h \le \alpha$

b) f *is continuous on the set* $C\left(\left[-h, 0\right], \mathbb{R}^n\right)$

c) f satisfy the Lipschitz condition, i.e. for any $\alpha > 0$ *there exists a Lipschitz constant* $L(\alpha) > 0$ *such that* $\| f(\varphi_1) - f(\varphi_2)\| \le L(\alpha)\|\varphi_1 - \varphi_2\|_h$, $\varphi_k \in C^1\left(\left[-h, 0\right], \mathbb{R}^n\right)$, *and* $\|\varphi_k\|_h \le \alpha,\ k = 1, 2.$

Then, for a given $t_0 \ge 0$ *and an initial condition* $\varphi \in C\left(\left[-h, 0\right], \mathbb{R}^n\right)$, *there exists* $\tau > 0$ *such that the system admits a unique solution* $x(t)$ *of the initial value problem (3), with the solution defined on the interval* $[t_0 - h, t_0 + \tau]$.

Definition 2. *The zero equilibrium* (*or zero solution*) *of the equation* (2) *is said to be stable if for any* $\varepsilon > 0$ *there exists* $\delta(\varepsilon) > 0$ *such that for every initial condition* $\varphi \in C\left(\left[-h, 0\right], R^n\right)$ *with* $\|\varphi\|_h < \delta(\varepsilon)$, *the inequality* $\left\|x\left(t; t_0, \varphi\right)\right\| < \varepsilon$ *take place for every* $t \ge 0$.

A stronger stability is introduced further.

Definition 3. *The zero solution of equation* (2) *is said to be asymptotically stable if it is stable and there exists* $\delta_0 > 0$ *such that if* $\left\|x_{t_0}\right\| \le \delta_0$, *for every* $\varepsilon > 0$ *there exist* $T(x_{t_0}, t_0, \varepsilon) > 0$ *such that* $\left\|x\left(t; t_0, x_{t_0}\right)\right\| < \varepsilon$ *if* $t > t_0 + T$. *If T does not depend of* x_{t_0}, *the stability is called equal asymptotic stability, if it is dependent only on* ε, *then it is called uniform asymptotic stability.*

Definition 4. *The zero solution of system* (2) *is said to be exponentially stable if there exist* $\Delta_0 > 0, \sigma > 0,$ *and* $\gamma \ge 1$ *such that for every* $t_0 \ge 0$ *and any initial function* $\varphi \in C\left(\left[-h, 0\right], R^n\right)$, *with* $\|\varphi\|_h < \Delta_0$, *the following inequality holds* $\left\|x\left(t, t_0, \varphi\right)\right\| \le \gamma \|\varphi\|_h\, e^{-\sigma\left(t - t_0\right)}$, $t \ge t_0$.

The exponential stability is even stronger than the asymptotic one, given the fastest decrease to zero of the perturbed solution, the exponential decrease.

These types of stability refer to *local* stability, they describe the behaviour of solutions that start in a limited neighbourhood of the equilibrium point.

Switched systems are part of the extensive class of hybrid dynamical systems. Such systems are characterized by the interaction between continuous and discrete systems (Liberzon, 2003), (Savkin & Evans, 2002). The field of hybrid systems is relatively new and with applicability in many real-world problems, so it is of increasing interest to the scientific world.

A switched system is an ensemble composed of m subsystems and a piecewise constant function $\sigma(t)$, called switching rule (or switching signal)

$$\dot{x} = f_{\sigma(t)}(x(t), u(t)),\ x(t_0) = x_0$$
$$\sigma : \mathbb{R}_+ \to \{1, ..., m\}.$$

The signal σ specifies the subsystem $\sigma(t)$ (also called *mode* in Thesis) operating at each time t. The signal is a discontinuous function at certain instants t in $\mathbb{R}_+$, called switching times, and is constant on every time interval between two consecutive switching times. Only a finite number of switchings occurs on any finite time interval. $\sigma(t)$ is a continuous function from the right everywhere: $\lim_{\tau \to t^+} \sigma(t) = \sigma(t)$ for each $\tau \geq 0$ (Liberzon, 2003), (Sun & Ge, 2005a). By $x(t)$ is noted the system state and by $u(t)$ the control input.

It is noteworthy that if the component subsystems are stable, this does not guarantee the stability of the entire switched system (Liberzon, 2003), (Benítez & Pérez, 2011). Based on the choice of control law, it can stabilize a switched system that has unstable subsystems (Wang, et al., 2019), (Yang, et al., 2014), (Dimirovski, et al., 2018), (Niculescu, 2001), or it can destabilize a switched system with stable subsystems (Cao, et al., 2019), (Wang, et al., 2016b), (Zhao, et al., 2017). Another specific property of this type of systems is that at a given time only one subsystem is active.

In this Thesis, the case of a hydraulic servomechanism is studied. This, by its nature, has structural switching, in other words the law of switching is not established a priori, the switching is based on changing the sign of a state which is practically impossible to be controlled. So we will talk about switched system with state-dependent and uncontrolled switching rule.

2. Critical case of equilibrium stability of a nonlinear dynamical system with structural switching and state delay

In this Chapter, a nonlinear system with state delay and switching is studied. The system presents a critical case for linear stability which makes it difficult to establish whether or not the system is stable according to Lyapunov's theory of stability by the first approximation. Thus, the problem will be developed in the framework of Malkin's approach (Malkin, 1966). Complete Lyapunov-Krasovskii functionals (Kharitonov, 2013) will be used.

Consider the general nonlinear switched time-delay system

$$\begin{aligned} \dot{x}(t) &= A_i x(t) + B_i x(t-h) + F_i\left(x(t), x(t-h), y(t)\right), t \geq 0 \\ \dot{y}(t) &= G_i\left(x(t), x(t-h), y(t)\right) \end{aligned} \tag{4}$$

where $x = (x_1, \ldots x_n)^T \in \mathbb{R}^n$, $y \in \mathbb{R}$ and for $i = 1, 2$, $A_i, B_i \in \mathrm{M}_n(\mathbb{R})$ ($n \times n$ matrices). F_i and G_i contain only powers of x_j, $j = 1, \ldots n$, of order greater than or equal to two, $F_i(0,0,y) = G_i(0,0,y) = 0$, $\forall y \in \mathbb{R}$, and for every $\delta > 0$ there exist $M_1(\delta)$ and $M_2(\delta)$ with the property $\lim_{\delta \to 0} M_1(\delta) = \lim_{\delta \to 0} M_2(\delta) = 0$ so whenever $\|x(t)\| \leq \delta$, $\|x(t-h)\| \leq \delta$, $\|y(t)\| \leq \delta$, the following inequalities take place

$$\begin{aligned} \left\|F_i\left(x(t), x(t-h), y(t)\right)\right\| &\leq M_1(\delta)\left(\|x(t)\| + \|x(t-h)\|\right) \\ \left\|G_i\left(x(t), x(t-h), y(t)\right)\right\| &\leq M_2(\delta)\left(\|x(t)\| + \|x(t-h)\|\right). \end{aligned} \tag{5}$$

System (4) has a form that anticipates the presence of a critical case for stability given the absence of the linear part in the equation for y. Therefore, a theorem for the critical case of equilibrium stability for the nonlinear switched time-delay system (4) will be stated and proved.

For the beginning, the attention is directed towards the study of the equilibrium stability for the first equation from (4)

$$\dot{x}(t) = A_i x(t) + B_i x(t-h) + F_i\left(x(t), x(t-h), y(t)\right) \tag{6}$$

where $y(t)$ is supposed to be bounded for all $t \geq 0$. Denote by $x(t; 0, \varphi)$ the solution of the system (6) that fulfils the relation $x\left(\theta; 0, \varphi\right) = \varphi\left(\theta\right), \theta \in \left[-h, 0\right]$. Define $x_t(0, \varphi)\left(\theta\right) := x\left(t + \theta; 0, \varphi\right), \theta \in \left[-h, 0\right]$, for $t \geq 0$. Consider $C\left([-h, 0], \mathbb{R}^n\right)$ the normed space of continuous functions with the standard uniform norm defined as $\|\varphi\|_h = \sup_{-h \leq \theta \leq 0} \|\varphi(\theta)\|$.

The above system can be rewritten in a compact, general form

$$\dot{z} = f_i(z_t) \tag{7}$$

with f_i functionals defined on $C\left([-h,0],\mathbb{R}^n\right)$.

Theorem 5. *Consider the nonlinear system (7) with f not indexed (i.e., no switching). Suppose* $f: C \to \mathbb{R}^n$ *maps every bounded set* C *in* $\mathbb{R}^n$ *and satisfies all the conditions for the existence and uniqueness of a local solution for (7). Also* $f(0)=0.$ *A necessary and sufficient condition for the asymptotic stability of the zero solution of (7) is the existence of a continuous differentiable functional* $V: C \to \mathbb{R}$ *so that* $\exists\, m, M, \psi : \mathbb{R}^+ \to \mathbb{R}^+$ *continuous increasing functions, with* $m(0)=M(0)=\psi(0)=0,$ *such that for the solutions of (7), for each* $t \geq 0,$

$$m\left(\|x(t)\|\right) \leq V(x_t) \leq M\left(\|x(t)\|_h\right)$$
$$\dot{V}(x_t) \leq -\psi\left(\|x(t)\|\right).$$

According to some consulted sources (Gu, et al., 2003), (Kharitonov, 2013), (Gu & Niculescu, 2003), (Fridman, 2014b), the above Theorem 5, often called Lyapunov-Krasovskii Stability Theorem, originated in Krasovskii's book (Krasovskii, 1959) (Theorems 31.1-31.3). The version written here is close to the one from (Kolmanovskii & Nosov, 1981), (Afanasiev, et al., 2003), (Gu & Niculescu, 2003) and (Kim, et al., 2008), the latter citing as source (Hale & Verduyn Lunel, 1993).

A functional $V\left(x_t\right)$ fulfilling the conditions of Theorem 5 is called a Lyapunov-Krasovskii functional (Kharitonov, 2013), (Kim, et al., 2008).

The model is constructed starting from the simple case of asymptotically stable linear systems

$$\dot{x}(t) = A_i x(t) + B_i x(t-h),\; i = 1,2. \tag{8}$$

Consider the Lyapunov-Krasovskii functionals $V_i(x_t),\; i=1,2$ (Kharitonov, 2013), (Kharitonov & Zhabko, 2003), (Kim, et al., 2008)

$$V_i(x_t) = V_{0,i}(x_t) + \int_{-h}^{0} (1+h+s)\|x(t+s)\|^2 ds \tag{9}$$

with $V_{0,i}$ defined by

$$\begin{aligned} V_{0,i}(x_t) &= x^T(t)U_i(0)x(t) + 2x^T(t)\int_{-h}^{0} U_i(-h-s)B_i x(t+s)ds \\ &+ \int_{-h}^{0}\left(\int_{-h}^{0} x^T(\theta+t)B_i^T U_i(s-\theta)B_i x(t+s)d\theta\right)ds \end{aligned} \tag{10}$$

A necessary and sufficient condition for the existence of the Lyapunov-Krasovskii functional (9) is that system (8) is asymptotically stable.

In the following, the mathematical model for the critical case of stability of the nonlinear system with delay and structural switching is constructed within the four properties stated in the paper (Kim, et al., 2008). Consider $i,\ j=1,2, i\neq j,$ the two possible modes the system can take.

Proposition 6. *There exist strictly positive numbers* α,β *so that for every* $t\geq 0,$ *the solution* x_t *of (6) verifies*

$$\alpha\left\|x(t)\right\|^2\leq V_i(x_t)\leq\beta\left\|x_t\right\|_h^2. \tag{11}$$

Proposition 7. *The derivatives of the Lyapunov-Krasovskii functionals (9) along the solution of the nonlinear systems (6) verify*

$$\dot{V}_i(x_t)\leq -w(x_t) \tag{12}$$

with w *positive definite and* $w(0)=0.$

Proposition 8. *There exists* $\mu>1$ *such that*

$$V_i(x_t)\leq\mu V_j(x_t) \tag{13}$$

for all x_t *solutions of the nonlinear systems (6) where* $i,\ j=1,2,\ i\neq j.$

Introduce now, following (Kim, et al., 2008), an hypothesis on the behaviour of the functionals V_i during consecutive switching times.

(H) For $i=1,2,$ *there exist constants* $c_i\in(0,1)$ *so that, for any pair of consecutive switching times of the i*th *mode,* $t_p<t_q$ *with the i*th *mode active in* t_p *and* t_q*, one has*

$$V_i(x_{t_q})-V_i(x_{t_p})\leq -c_iV_i(x_{t_p}) \tag{14}$$

where x_t *is the solution of the system (6).*

Theorem 9. *Suppose* $\left\|y(t)\right\|\leq\delta,\ \forall t\geq 0.$ *Then the zero solution of the switched system (6) is asymptotically stable.*

Definition 10. *The switching rule* σ *is defined as stable if there exists* $\varepsilon>0$ *such that if matrices* $\hat{A}_i$ *and* $\hat{B}_i$, $i=1,2,$ *satisfy* $\left\|\hat{A}_i-A_i\right\|<\varepsilon,\ \left\|\hat{B}_i-B_i\right\|<\varepsilon,$ $\hat{\delta}$ *verify* $\left|\hat{\delta}-\delta\right|<\varepsilon$ *and the linear systems (8) are stable, the Lyapunov-Krasovskii functionals (9) verify (H) when* x_t *is a solution of the active system similar to (6) for some* $\hat{F}_i$ *that verifies (5) for an* $\hat{M}\left(\hat{\delta}\right)$ *with* $\left|\hat{M}\left(\hat{\delta}\right)-M\left(\delta\right)\right|<\varepsilon.$

Theorem 11. *Suppose that (H) holds and the switching rule* σ *is stable. Then the zero solution of the switched system (4) is stable.*

3. Equilibrium stability of a nonlinear feedback system with structural switching and delay on control

The main objective is to propose a solution to the complex problem raised by the stability of the equilibrium point of a realistic nonlinear feedback system with input (or control) delay and structural switching. The mathematical framework is that of a predictive feedback control based on a LQR (Linear Quadratic Regulator) synthesis and of the multiple quadratic type Lyapunov-Krasovskii functional to decide on stability. The starting point for this study was given by the challenges generated by the behaviour of the mathematical model of EHS. Therefore, the theory developed in this Chapter is applied to the case of an EHS.

Consider the following linear time invariant system with delay on control and without switching

$$\begin{aligned} &\dot{x}(t) = Ax(t) + B_c u(t-h);\, x(0) = x_0 \neq 0, \\ &u(t) = u_0(t), -h \leq t \leq 0,\, h > 0, \end{aligned} \tag{15}$$

with $A \in \mathbb{R}^{n\times n}$, $B_c \in \mathbb{R}^{n\times 1}$ and the pair (A, B_c) complete controllable, or at least stabilizable (Kwon & Pearson, 1980), (Kwakernaak & Sivan, 1972). The aim is to study the stability of the zero point equilibrium of the system (15) for $t > 0$, with $u(x(t))$ the state feedback control and the initial conditions given by $u_0(.)$, x_0. A perturbation of the zero equilibrium point will be denoted as x_0. Here the control u is assumed to be a locally integrable function: $u \in \mathbb{L}^1_{loc}[-h, \infty) \to \mathbb{R}$.

If the system (15) would be without delay, then the feedback control law would be given, for example, by a state feedback $u(t) = Kx(t)$, $K \in \mathbb{R}^{1\times n}$ provided using LQR algorithm. When the delay appears on control, the things get more complicated. One of the classic approaches used in this problem is the method of the predictive feedback control which can be briefly described as follows: find a predictive control law $u(t-h) = Kx(t)$, in other words $u(t) = Kx(t+h)$ to stabilize the system with control delay. Therefore, a prediction of the state would be needed.

Proposition 12. *Consider system (15) with the pair* (A, B_c) *controllable, or at least stabilizable.*

a) A state predictor for system (15) is given by

$$x_p(t) := x(t+h) = e^{Ah}x(t) + \int_{-h}^{0} e^{-As}B_c u(t+s)\,ds. \tag{16}$$

b) Applying state predictive feedback control $u(t) = Kx(t+h)$*, system (15) is replaced by the compensated system*

$$\dot{x}(t) = Ax(t) + A_d x(t-h) + B_c K\int_{-h}^{0} e^{-As}B_c u(t+s-h)ds; A_d := B_c K e^{Ah}. \tag{17}$$

Consider the nonlinear feedback system with structural switching and state delay transferred above from the actuator

$$\begin{gathered}\dot{x}(t) = A_i x(t) + A_{di}x(t-h) + B_c K_i\int_{-h}^{0} e^{-A_i s}B_c u_i(t+s-h)\,ds + F_i\left[x(t)\right],\\ A_{di} := B_c K_i e^{A_i h}, i = 1,\dots,m.\end{gathered} \tag{18}$$

Systems (18) are a nonlinear extension of system (17) in which the switched structure and the remainders of order one $F_i(x(t))$ obtained following the Taylor series development around the origin are added.

Theorem 13. *Consider systems (18) with*

a) A_i *are Hurwitz matrices, therefore there are symmetric positive definite matrices* P_i *satisfying Lyapunov matrix equations* $A_i^T P_i + P_i A_i = -Q_i$ *for some symmetric positive definite matrices* Q_i *and*

b) A_{di} *are small enough matrices, specifically* $\|P_i A_{di}\| < \lambda_{\min}(Q_i)/2$*, therefore there are* $\omega_i > 0$ *such that* $\|P_i A_{di}\| \le \omega_i < \lambda_{\min}(Q_i)/2$.

For each $i = 1,\dots,m$ *in (18), the following Lyapunov-Krasovskii functional is considered*

$$V_i(x_t) = x^T(t)P_i x(t) + \omega_i\int_{t-h}^{t}\|x(s)\|^2\,ds. \tag{19}$$

Then the conditions (11) and (12) are fulfilled for all $i = 1,\dots,m$*, as long as the functions* $\psi_i(\|x(t)\|) := \left\{\lambda_{\min}(Q_i) - 2\left[\omega_i + \lambda_{\max}(P_i)(M_i\|x\| + N_i)\right]\right\}\|x(t)\|^2$ *are positive.*

Proposition 14. *The Lyapunov-Krasovskii functional (19) fulfils condition (13).*

Theorem 15. *A sufficient condition of asymptotic stability of the zero solution of systems (18) is that the functionals* $V_i(x_t)$*,* $i = 1,\dots,m$*, given by (19) meet conditions (11), (12) and Hypothesis (H).*

4. Applications to hydraulic servomechanisms. Numerical simulations

In the following, the theory developed in Chapters 2 and 3 is applied to hydraulic servomechanisms. Therefore, in Section 4.1 is given a short description of the mechanohydraulic and electrohydraulic servomechanisms and the mathematical models that characterize them are obtained. Also, mathematical models containing time-delay are proposed. In Section 3.2 is given an application of the theorem proved in Chapter 2, namely a theorem for a critical case of a nonlinear switched system with state delay. Section 4.3 represents an application of the theorem for equilibrium stability of a nonlinear feedback system with structural switching and time-delay on control proved in Chapter 3. Moreover, a method of obtaining a predictive feedback control law is presented in Section 4.4.

Mathematical models of mecanohydraulic and electrohydraulic servomechanisms

Current technologies involve working with machines capable of lifting extremely heavy weights, thus involving high forces, or performing movements at high speeds and precision. For such operations, industrial machines are equipped with hydraulic servomechanisms of a mechanical nature, and we are talking about mechanohydraulic servomechanism (MHS), or of an electrical nature as is the case with electrohydraulic servomechanisms (EHS). A brief description of the mathematical models of MHS and EHS is presented in the thesis.

The complexity of the hydraulic servomechanisms and their dynamics could be sources of time-delays occurrence: inertia of the moving components and of the controlled load, constitutional nonlinearities of the servomechanism, as is the overlap in the spool valve, for instance, complex dry friction between moving and fixed parts of the hydraulic cylinder (see LuGre model (Olsson, et al., 1998)), delays on the command line from the transducer, the delay given by the pilot's reaction time (Toader & Ursu, 2014), delay generated by the computing unit to synthesise the control law.

Different mathematical models with time-delay are presented for EHS and MHS: mathematical models with structural switching and with time-delay on the servovalve state, on control, or introduced on state due to the presence of friction. These models are the starting point for the substantiation of the stability analysis and for the understanding of the consequences of the time delay presence in the dynamic equations.

Proposition 16. *A primary mathematical model with time delay for the EHS is described by the following time-delay autonomous, nonhomogeneous, differential equation*

$$\dot{x}_1(t) + kx_1\left(t - h_0\right) = \frac{k}{k_T} u_{ref}\left(t - h_0\right) \tag{20}$$

where h_0 *is an equivalent time delay containing inertia and viscous load effects in hydrocylinder and electrohydraulic servovalve,* $h_0 = h + h_e$.

Proposition 17. *The EHS described by the equation with time delay (20) is stable (i.e. has roots with negative real parts) if* $kh_0 < \frac{\pi}{2}$.

Proposition 18. *A primary mathematical model with time delay for the MHS is described by the time-delay differential equation*

$$\dot{x}_1(t) + kx_1\left(t - h_0\right) = kx_r\left(t - h_0\right). \tag{21}$$

Application to electrohydraulic servomechanisms

The following mathematical model of EHS represents the splitting of the system EHS into two subsystems with respect to the sign of the servovalve sign (Tecuceanu, et al., 2019), (Halanay & Ursu, 2009), (Halanay & Ursu, 2010), (Balea, et al., 2010), (Ursu, et al., 2013), (Halanay, et al., 2009), (Halanay, et al., 2004), (Ursu, et al., 2006), (Ursu & Ursu, 2007), (Balea, et al., 2010)

$$\begin{gathered}\dot{y}_1 = y_2;\ \dot{y}_2 = -\frac{k}{m}y_1 - \frac{f}{m}y_2 + \frac{S}{m}y_3 - \frac{S}{m}y_4;\ \dot{y}_3 = \frac{B}{V_0 + Sy_1 + S\hat{x}_1}\left(Cy_5(t-h)\sqrt{p_s - y_3 - \hat{x}_3} - Sy_2\right) \\ \dot{y}_4 = \frac{B}{V_0 - Sy_1 - S\hat{x}_1}\left(-Cy_5(t-h)\sqrt{y_4 + \hat{x}_4} + Sy_2\right);\ \dot{y}_5 = -\frac{k_{SV}k_T}{\tau_{SV}}y_1 - \frac{1}{\tau_{SV}}y_5(t-h).\end{gathered} \tag{22}$$

The characteristic equation $\det\left(sI - A_1 - B_1 e^{-sh}\right) = 0$ has an eigenvalue zero, meaning that it is a critical case of stability and one cannot decide about the stability of this equilibrium using the first approximation. Consequently, the system (22) has to be brought to the form where the Lyapunov-Malkin type Theorem 11 can be applied. The new system is given by

$$\begin{aligned}\dot{\eta} &= G_1\left(\eta, \xi_2, \xi_3, \xi_4, \xi_5(t-h)\right) \\ \dot{\xi}_2 &= -\frac{f}{m}\xi_2 + \left(\frac{S}{m} + a_3\frac{k}{m}\right)\xi_3 + \left(a_4\frac{k}{m} - \frac{S}{m}\right)\xi_4 \\ \dot{\xi}_3 &= a_{32}\xi_2 + b_{35}\xi_5(t-h) + \overline{g}_3 - c_3\dot{\eta} \\ \dot{\xi}_4 &= a_{42}\xi_2 + b_{45}\xi_5(t-h) + \overline{g}_4 - c_4\dot{\eta} \\ \dot{\xi}_5 &= -a_3a_{51}\xi_3 - a_4a_{51}\xi_4 - \frac{1}{\tau_{SV}}\xi_5(t-h)\end{aligned} \tag{23}$$

and has the form specific to the Lyapunov-Malkin type theory

$$\begin{aligned}
&\dot{\xi}(t) = A_i\xi(t) + B_i\xi(t-h) + F_i\left(\xi(t), \xi(t-h), \eta(t)\right) \\
&\dot{\eta}(t) = G_i\left(\xi(t), \xi(t-h), \eta(t)\right), \quad i = 1, 2.
\end{aligned} \tag{24}$$

Theorem 11 ensures the stability of the solution for system (24), if the linear components

$$\dot{\xi}(t) = A_i\xi(t) + B_i\xi(t-h), \;\; i = 1, 2 \tag{25}$$

fulfil the stability conditions and the Lyapunov conditions.

Theorem 19. (Hu & Wang, 2002)**,** (Xu, et al., 2014)**.** *The linear delayed system (24) is delay-independent stable if and only if the following two conditions hold true:*
a) the characteristic polynomial $P(s) + Q(s)$, corresponding to the case $h = 0$, has only roots with negative real parts;
b) the polynomial $F(\omega) = |P(i\omega)|^2 - |Q(i\omega)|^2 = 0$ has no real roots ω other than zero.

Definition 20. [(Kharitonov, 2013), Definition 2.6]. *The Lyapunov condition for systems (25) refers to the interdiction that their characteristic equations have symmetric roots with respect to the origin*.

The calculated theoretical value $h_{\max}$ is remarkably well validated by the numerical simulation of the system (22). The critical value for the delay, beyond which the system becomes unstable was found at $h_{\max} = 0.0116$ s.

Equilibrium stability of the mathematical model of EHS. Numerical simulations

The mathematical model of the EHS is described by a nonlinear system with five differential equations, four of them define the valve-actuator-load system, and the fifth represents a first order dynamics of the EHSV. Furthermore, it shows a switching nonlinearity due to constructive directional changes in the dynamic of the spool valve ports opening

$$\begin{aligned}
&x_5 \geq 0: \\
&\dot{x}_1 = x_2;\, \dot{x}_2 = -\frac{k}{m}x_1 - \frac{f}{m}x_2 + \frac{S}{m}x_3 - \frac{S}{m}x_4;\, \dot{x}_3 = \frac{B}{V_0 + Sx_1}\left(Cx_5\sqrt{p_s - x_3} - Sx_2 + k_l\left(p_s - 2x_3\right)\right) \\
&\dot{x}_4 = \frac{B}{V_0 - Sx_1}\left(-Cx_5\sqrt{x_4} + Sx_2 + k_l\left(p_s - 2x_4\right)\right);\, \dot{x}_5 = -\frac{x_5}{\tau_{SV}} + \frac{k_{SV}}{\tau_{SV}}u_1\left(x\left(t-h\right)\right)
\end{aligned} \tag{26}$$

$$\begin{aligned}
&x_5 < 0: \\
&\dot{x}_1 = x_2;\, \dot{x}_2 = -\frac{k}{m}x_1 - \frac{f}{m}x_2 + \frac{S}{m}x_3 - -\frac{S}{m}x_4;\, \dot{x}_3 = \frac{B}{V_0 + Sx_1}\left(Cx_5\sqrt{x_3} - Sx_2 + k_l\left(p_s - 2x_3\right)\right) \\
&\dot{x}_4 = \frac{B}{V_0 - Sx_1}\left(-Cx_5\sqrt{p_s - x_4} + Sx_2 + k_l\left(p_s - 2x_4\right)\right);\, \dot{x}_5 = -\frac{x_5}{\tau_{SV}} + \frac{k_{SV}}{\tau_{SV}}u_2\left(x\left(t-h\right)\right); \\
&C := c_d w\sqrt{\frac{2}{\rho}}.
\end{aligned} \tag{27}$$

The initial conditions associated with systems (26)-(27) are as follows: $x_i(0) = x_{0,i} \neq 0$, $u_i(t) = u_{0,i}(t)$, $-h \le t \le 0$, $h > 0$, $i = 1, 2$. Two types of leakage are introduced, namely, one internal in spool valve of EHSV and one external in hydrocylinder. In the absence of any leakage, the Jacobian matrices have two critically zeros, while the model as in (Meritt, 1976), with both internal and external leakages, has singularity only in the case of $x_1 = 0$.

Let A_1 and A_2 be the Jacobian matrices calculated in zero for both cases $x_5 \ge 0$ and, obvious, $x_5 < 0$

$$\boldsymbol{A}_1 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{k}{m} & -\frac{f}{m} & \frac{S}{m} & -\frac{S}{m} & 0 \\ 0 & -\frac{BS}{V_0 + S\hat{x}_{1,1}} & -\frac{B}{V_0 + S\hat{x}_{1,1}}\left(\frac{C\hat{x}_{5,1}}{2\sqrt{p_s - \hat{x}_{3,1}}} + 2k_l\right) & 0 & \frac{BC\sqrt{p_s - \hat{x}_{3,1}}}{V_0 + S\hat{x}_{1,1}} \\ 0 & \frac{BS}{V_0 - S\hat{x}_{1,1}} & 0 & -\frac{B}{V_0 - S\hat{x}_{1,1}}\left(\frac{C\hat{x}_{5,1}}{2\sqrt{\hat{x}_{4,1}}} + 2k_l\right) & -\frac{BC\sqrt{\hat{x}_{4,1}}}{V_0 - S\hat{x}_{1,1}} \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_{SV}} \end{pmatrix} \quad (28)$$

$$\boldsymbol{A}_2 = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ -\frac{k}{m} & -\frac{f}{m} & \frac{S}{m} & -\frac{S}{m} & 0 \\ 0 & -\frac{BS}{V_0 + S\hat{x}_{1,2}} & \frac{B}{V_0 + S\hat{x}_{1,2}}\left(\frac{C\hat{x}_{5,2}}{2\sqrt{\hat{x}_{3,2}}} - 2k_l\right) & 0 & \frac{BC\sqrt{\hat{x}_{3,2}}}{V_0 + S\hat{x}_{1,2}} \\ 0 & \frac{BS}{V_0 - S\hat{x}_{1,2}} & 0 & \frac{B}{V_0 - S\hat{x}_{1,2}}\left(\frac{C\hat{x}_{5,2}}{2\sqrt{p_s - \hat{x}_{4,2}}} - 2k_l\right) & -\frac{BC\sqrt{p_s - \hat{x}_{4,2}}}{V_0 - S\hat{x}_{1,2}} \\ 0 & 0 & 0 & 0 & -\frac{1}{\tau_{SV}} \end{pmatrix}. \quad (29)$$

The matrix of control influence B_c is a column vector with the first four elements zero and with the fifth element equal with k_{SV} / τ_{SV}. The pairs (A_i, B_c), i =1, 2, are not completely controllable, but are stabilizable, including in $x_0 = 0$ which is the most vulnerable equilibrium point of the EHS system (Guillon, 1972). In fact, the two matrices A_i (28) and (29) are Hurwitz matrices. The control law synthesis is made through an LQR procedure and it concerns the pairs of matrices (A_i, B_c), $i = 1, 2$.

The equilibrium vector point corresponding to the subsystem with the state variable $x_5 \ge 0$ is obtained by choosing a value for $\hat{x}_{1,1}$ and the others are found according to the relations $\hat{x}_{2,1} = 0$, $\hat{x}_{3,1} = p_s/2 + kx_{0,1}/(2S)$, $\hat{x}_{4,1} = p_s/2 - kx_{0,1}/(2S)$, and $\hat{x}_{5,1}$ is found as a solution of $Cx_{5,1}\sqrt{(p_s - kx_{0,1}/S)/2} - k_l kx_{0,1}/S = 0$. Thus, one obtains the following values: $\hat{x}_{1,1} = 5\times10^{-3}$m, $\hat{x}_{2,1} = 0$ m/s, $\hat{x}_{3,1} = 112.5\times10^5$ N/m^2, $\hat{x}_{4,1} = 97.5\times10^5$ N/m^2, $\hat{x}_{5,1} = 0.0018\times10^{-3}$ m, with $u_1 = 0.0925$. The eigenvalues of the matrix A_1 (28) in open loop, in accordance with the values of the chosen equilibrium point, are: $\lambda_{1,2} = -97.4 \pm 1726.5i$, $\lambda_3 = -0.3$, $\lambda_4 = -90$, and $\lambda_5 = -131.2$. The closed loop eigenvalues of the matrix A_1 (28) are $\lambda_{1,2} = -97.4 \pm 1726.5i$, $\lambda_3 = -10.2$, $\lambda_4 = -90$, $\lambda_5 = -130.8$. The LQR control law synthesis was obtained choosing the weighting matrices Q_J, as zero matrix excepting Q_J (1,1) = 1 and $R_J = 0.0025$, thus obtaining the feedback gain as K_1 = [6.1005 0.0002 0.0008 −0.0006 3.6293]. The same procedure is followed for the second subsystem characterized by $i = 2$.

To verify the stability conditions described in Theorem 13, so to fulfil the inequality $\psi_i\left(\left\|x(t)\right\|\right) \geq 0$, the following parameters are required: x_{0i}, k_l, h, M_i, $\left\|x(t)\right\|$, $\left\|R_i(t)\right\|$, N_i, $\lambda_{\min}(Q_i)$, $\lambda_{\max}(P_i)$, ω_i. Since matrices Q_i and P_i are correlated due to the Lyapunov equation, ensuring a positive value of the expression $\lambda_{\min}(Q_i) - 2\left[\omega_i + \lambda_{\max}(P_i)\left(M_i\left\|x\right\| + N_i\right)\right]$ is extremely difficult.

It is highlighted the influence of the delay on the actuator on the evolution of the system. Furthermore, a delay threshold nearly $h = 0.1$ s is identified up to which the system remains stable. For $h = 0.096$ s, there is a pole in discrete time at the stability limit, $z = 0.99995$. For $h^* = 0.1$ s, the pole becomes unstable $z = 1.0002346$.

Model discretization and numerical simulations for a linear system with actuator delay

In the following, a numerical-analytic method for the synthesis of a predictive control law is proposed. This is applied to the linearized mathematical model (18) of the electrohydraulic servomechanism with delay on control and structural switching analysed in the previous Subchapter 3.2. In other words, the problem of finding a feedback control law is formulated and solved in order to ensure the equilibrium stability of the dynamic system influenced by the presence of the delay.

The mathematical model (18) implies the existence of an integral term that contains the history of control. Practically, in order to find the solution of the DDE (18), besides the given initial condition, it is necessary to know all the previous control laws and to be included in the calculation. This procedure is not very easy to perform and involves a discretization of the model to compensate for the effects of delay. Thus, the integration of the system requires the replacement of the integral with a sum by dividing the length *h* of the integration interval into a proper number of *k* sampling periods *T*. The results of the numerical simulations are presented highlighting the maximum delay until the system remains stable.

Consider the following linear invariant system with delay on control as input

$$\dot{x}(t) = Ax(t) + Bu(t-h),\ x(0) = x_0 \tag{30}$$

and the control law

$$u(t-h) = -Kx(t). \tag{31}$$

The sources of the delay that may occur on the control could be:

1. an intrinsic characteristic of the plant, or
2. the necessary time to compute the control law when this is numerically synthesize; in this case, the delay is equal with the sampling period.

The working hypothesis is that the feedback matrix corresponding to law (31) has been determined such that the matrix $A-BK$ is stable. Moreover, the closed loop system without delay meets certain performance criteria. If a delay occurs in the system and the control law does not take this into account, undesirable effects may occur in the system dynamics such as performance degradation or even destabilization. Therefore, two directions of study can be approached:

(D_1) determining a maximum value of the delay $h_{\max}$ beyond which the system (30) is unstable using the control law (31)

(D_2) synthesizing a control law so as to counteract the effects of the delay; in other words, for $t>h$ the closed loop system behaves like the following system $\dot{x}=(A-BK)x,\ x(h)=e^{Ah}x_0$. This condition is fulfilled by the predictive control law in the following Proposition.

Proposition 20. *The feedback control law for the linear system (29) which fulfils the objective (D_2) stated above is given by the following predictive feedback control law*

$$u(t)=-Kx(t+h)=-K\,e^{Ah}x(t)-K\int_{t-h}^{t}e^{A(t-s)}Bu(s)ds. \tag{32}$$

Proposition 21. *The feedback control law* (31) *is computed according to the discretised form*

$$u(n)=-K\ A_D^k x(n)-K\sum_{i=n-k}^{n-1}A_D^{n-1-i}\ B_D\,u(i) \tag{33}$$

$$t:=nT,\ n=0,1,2,\ldots,\ h=kT,\ A_D:=e^{AT},\ B_D:=A^{-1}[e^{AT}-I]B.$$

Conclusions

C.1. General conclusions

The main objective of this PhD Thesis was to study the local stability of the equilibrium points of some dynamical systems described by differential equations with delay and structural switching. Stability is understood as local stability, so stability for sufficiently small disturbances of equilibrium. Moreover, the stability of the servomechanism equilibrium as a stabilization system is equivalent to the stability of the servomechanism system as a tracking system.

The secondary objectives were to prepare the mathematical framework and to prove Propositions and Theorems on stability and, also, to validate the results through numerical simulations and real applications in engineering. The applications are related to the mechanohydraulic servomechanism and, especially, to the electrohydraulic servomechanism, both used in aviation for the control of the primary flight commands of the aircraft, in this case for the control of the ailerons.

Comparing the approaches in Chapters 2 and 3, some conclusions can be mentioned. The two mathematical models used are in fact related, in the sense that the delay on the control can be seen as a delay on the state of the servovalve and vice versa. The results in Chapter 2 have the advantage of sufficiently less restrictive stability conditions and the disadvantage, in the absence of adequate control, of the existence of a stability threshold in the presence of delay. The results from Chapter 3 have the advantage of an adequate control, of predictive type, which totally compensates the presence of the delay ensuring a stability of the equilibrium for sufficiently large disturbances and, also, for sufficiently large delays. The disadvantage is given by the high constraints of the stability conditions, conditions that are sufficient, not necessary.

The numerical simulations validated the analytical calculations in the sense that the maximum delay for which the switched system remains stable, calculated analytically, is basically the same as that found by numerical simulations, both in continuous time and in discrete time, in the presence of a simple LQR control, namely the delay $h_{\max} \simeq 0.1\,\mathrm{s}$. For control with predictive feedback, the delayed system behaves practically in the same way as the system without delay, both systems supporting sufficiently large disturbances of the equilibrium.

C.2. Perspectives of further development

The results obtained in this Thesis could be applied in different fields of engineering where the time-delay has an important effect in the dynamics of the system regarding the stability or the control law synthesis. Therefore, the mathematical model is improved so that it accurately describes the behaviour of the controlled object taking into account the history of its dynamics.

The current trend in the field of applied mathematics and engineering is to approach and study switched systems. Thus, the results from this Thesis follow this direction and offer a starting point for the development of new stability studies in which it is important to ensure the stability of the entire system composed of stable or unstable subsystems. A necessary and useful study is to find less conservative stability conditions taking into account that the Lyapunov-Krasovskii paradigm impose very constraining conditions.

Another important development horizon is to propose and realize practical models in the laboratory in order to validate the results obtained in the Thesis. In aerospace engineering, one meaningful objective is that of setting the flight envelope which ensures a safe flight beyond the limit of flutter occurrence. Flutter is a severe aeroelastic dynamic phenomenon consisting in self-sustaining unstable oscillations whose amplitude grows strongly in a short time accumulating energy in the structure. It affects the primary flight surfaces of an airplane since aeroelastic forces are developed at this level. It is a complex and difficult process to study. In recent years, various active flutter control methods have been approached using different kinds of actuators. An interesting and realistic solution to increase the flight envelope could be given by considering the time-delay on the line transducer-implemented control law-actuator.

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