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PHD THESIS SUMMARY

Design and optimization of structural components topology

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Keywords: design, factorial optimization, topological optimization, lattice, finite element method, modal analysis, homogenized material, additive manufacturing

Introduction

(i) Object of doctoral research

Optimization is an important process in many areas of business and is achieved by choosing the best of two or more possible solutions. Subjectivity, however, has an impact on the search for the optimal solution and is more pronounced the more possible solutions there are. As the range of possible solutions increases with the technical and economic progress, it becomes more difficult to choose the optimal one. Under these circumstances, the optimal solution must be objectivized, especially when the range of possible solutions increases indefinitely. In most cases, the range of possible solutions is infinite, and out of the infinity of possible solutions, one is the best solution for a given purpose, i.e. the optimal one.

Nowadays, it is considered that a rational solution to an optimization problem can only be obtained if the multiplicity of conditioning factors is equally taken into account. In this respect, it is noted that the elements to be optimized cannot be considered independently of the construction of which they form part, since there is an obvious interdependence between them and the rest of the construction, an interdependence which must be expressed in the optimization process. It is also noted that, at least at the present stage, it is not possible to carry out an overall optimization leading to a general solution capable of satisfying all the aspects and requirements imposed, and therefore the research being undertaken is aimed only at optimizing certain technical, technological and economic aspects.

Although the strictly mathematical solution of the optimization problem offers remarkable advantages, such an approach is not always acceptable, as it requires consideration of many objective and subjective factors. These can have contradictory characteristics and influences on the optimization, depending on the conditions of the structure in question, which is why less rigorous optimization methods are now accepted.

The optimization process is concerned both with the design, the overall composition and shape of the structure and the dimensioning of its components. In order to simplify problem solving, the structure's composition and general geometrical parameters are determined in advance, leaving only the physical and mechanical parameters to be optimized, such as the distribution of material in the components of the structural frame. In this case, certain dimensions defining the cross-sections of the elements and their geometrical characteristics are used as optimization parameters.

In general, although the decision variables are independent parameters, there are situations where the variables cannot be considered independent. The latter correspond to the case when the variables satisfy either certain condition equations (specific to the effort or displacement method) or relations expressing mechanical equilibrium at different stages of work. In these situations, the design variables are basically constrained conditions and will be considered in the optimization process as such.

The decision variables represent real numerical quantities to be determined when designing a structural frame. They are independent quantities that describe certain specific aspects of the problem, such as: geometrical characteristics of the component elements, configuration of the frane (size of spans, ratio of height to span), physical and mechanical properties of the materials from which the frame is made.

These parameters can vary either continuously or discretely, leading thus to an infinite or finite number of possible combinations.

Reducing the size of the optimization problem is possible by using standardized and modular elements in the composition of the structural frames and by imposing certain design considerations.

This eliminates the variation in the number of these parameters that characterize the cross-sections of the structural components, as they can have a limited number of values, which leads to a reduction in the number of combinations of them.

(ii) The novelty and importance of the doctoral research

The simplification of the optimization problem consists, at the present stage, in the schematization of real situations by introducing simplifying assumptions of a covering nature. With this in mind, some of the studies carried out to date have been applied in practice, as they have been completed in the form of algorithms and computer programs.

The analytical (classical) optimization methods can be applied to continuous or variable objective functions. These methods become impossible to apply as the decision variables are equality or inequality constraints and the size of the problem increases. For this reason, analytical methods are only applicable for solving simple optimization problems with small numbers of decision variables.

Numerical optimization methods are applicable for optimization problems with objective functions and complex constraints and large number of decision variables. Methods in this class are based on planned numerical experiments in which one advances step by step towards the desired extrema of the objective function by successive improvements of the function values.

Thus, numerical optimization procedures are used when analytical procedures are either inefficient or laborious and difficult (solving a large number of equations involved). Also, numerical procedures for testing the optimum are useful when the analytical expression of the performance function is not known, but experimental possibilities exist and the performance can be measured in the presence of known constraints. Numerical testing of the optimum leads to the approximate location of the optimum point, the approximation error being dependent on the test step and the test method involving different degrees of convergence.

Mathematical modelling is an important step in the algorithmic optimization process. It aims to identify mathematical relationships that describe the process to be optimized. The mathematical modelling step is not necessary when experimental optimization is carried out by modifying the factors influencing the parameters of the system being optimized according to a certain design in order to identify the optimal solution. The need to develop a mathematical model of the system when solving an optimization problem stems from the fact that, regardless of the optimization criterion chosen, its expression depends on the variables of the system.

A numerical method widely used today concerns the topological optimization dedicated to lattice structures. This method combined with the additive manufacturing process allows parts with a complex custom shape to be made, while providing precise control of the parts’ internal geometry. This topic is studied in detail in the present paper.

After designing the preliminary geometric model, the finite element method is used both to solve the topological optimization analysis resulting in the optimal lattice structure and to predict the behavior of the lattice structure under various mechanical stresses and to test several types of lattice cells. Thus, the finite element analysis is useful for the optimization of lattice structures.

The accuracy of this analysis depends on: the material properties assigned to the 3D model, the size of the elements, the geometry of the lattice cell and the relative density of the part. Analyzing the results obtained, it was observed that the relative density can influence the elasticity of the structure and that the finite element analysis can be used to evaluate the behavior of lattice structures.

The dimensional and mechanical properties of the lattice structures have been improved. It was found that an important factor that can influence the mechanical properties of lattice structures is the type of cell used. This has a direct effect on the stiffness of the structure. Also, the dimensional accuracy plays an important role in obtaining adequate mechanical properties of lattice structures. The correct choice of the additive manufacturing process parameters leads to a better quality of the lattice structures.

(iii) The purpose and objectives of the doctoral research

The main goal of the PhD thesis is to present an efficient experimental and numerical analysis methodology dedicated to the optimization of the topology of structural components, in order to improve their structural performance, contributing to the extension of the use of lattice structures in different applications of scientific or industrial interest.
In order to achieve the stated aim, the objectives of the PhD thesis can be grouped into two categories:

1. To conduct numerical research to provide information on the behavior of lattice structures under modal vibrations:
  1.1. to design and develop numerical models that describe as accurately as possible the response to modal vibrations of the lattice structures;
  1.2. to carry out a comparative study of several types of lattice cells in order to determine the optimum cell type;
  1.3. to use the homogenization method in numerical simulations for the study of lattice structures;
  1.4. numerical simulation of the additive manufacturing process;
  1.5. to use the factorial optimization methods together with the finite element method for determining the behavior of metallic structures under static and dynamic stresses.

2. To conduct experimental research to determine the eigenfrequencies specific to lattice structures:
  2.1. 3D printing of both the unoptimized and the optimized model (lattice model) using the SLA additive manufacturing process;
  2.2. determining the eigenfrequencies for the two models, both when they are fixed and when they are free;
  2.3. comparative analysis of the structural performance of the two models, based on the frequencies obtained from modal vibration tests;
  2.4. study of the influence of different geometrical, additive manufacturing and test parameters on the modal response of the lattice structures;
  2.5. comparative analysis of the numerical and experimental results.

(iv) Organization of the PhD Thesis

The PhD thesis is structured on eight chapters and an appendix, the content of which is briefly presented in the following pages.

Chapter 1 presents a brief history of DOE methods and their applicability in engineering. The knowledge necessary for their use in industry is mentioned as well as the principles that underlie them. A general procedure for performing the experiments is also briefly presented.

In the first part of Chapter 2 the most important experimental designs used in factorial methods are described. The experimental designs are the basis of the strategy for planning experimental trials in order to obtain useful and accurate results.

The Taguchi method is then presented, which has as its main objective the deviation of an output variable from the nominal value, the basic idea being that the minimization of the variation of the results is the main means of improving the quality of a product. Another important topic presented in Chapter 2 is the response surface methodology. This methodology is based exclusively on numerical simulation and provides the approximate values of the output parameters, from the entire optimization domain analyzed, without the need to perform a complete solution.

Chapter 3 begins with details of the concepts and structure of the optimization process, with a focus on structural optimization. The methodology for assessing the design under optimization is also briefly presented. Another topic mentioned in Chapter 3 is about the use of the finite element method in structural analyses. At the end of the chapter, important notions about the numerical correlation of parameters in the optimization process are presented, such as examples of correlation methods or sensitivity diagram.

In Chapter 4 fundamental notions related to topological optimization are presented, such as methods used for setting up topology optimization problems or various practical considerations. A short example is also presented.

In Chapter 5 a case study is presented which deals with improving the modal response for a mounting bracket using lattice topology optimization. The objective of this study was to obtain a lattice model whose fundamental frequency is maximized. In the first stage a custom topological optimization for lattice structures was performed. Once an optimal lattice model was obtained, a modal analysis was performed using the model itself. Although results were obtained, it was found that it is difficult to generate and then solve a discretized model obtained directly on a lattice structure. For this reason, an equivalent model obtained by the homogenization method was created which allows the generation and then the solution of the discretized model much faster. In this chapter a comparative study for several lattice cell types has also been carried out. Finally, the results were compared.

Chapter 6 is dedicated to the additive manufacturing process. Both the basic technologies of rapid prototyping and the advantages of 3D printing over traditional processes are presented. Emphasis is placed on the stereolithography process because the lattice model used in the experimental study was obtained by this printing method. In the second part of Chapter 6, another case study is presented, which consists of the numerical simulation of the additive manufacturing process of the lattice model studied.

In Chapter 7 a case study is presented which deals with the verification of the modal response for a mounting bracket using numerical simulations and experimental tests. As in Chapter 5, the objective of this study was to obtain a lattice model, this time reduced in size from the first case, whose fundamental frequency is maximized. In the first stage, a custom topological optimization for lattice structures was performed and a cubic cell was used. After an optimum lattice model was obtained, an equivalent model was created based on the homogenization method and this was subsequently used in the numerical study. The lattice model resulting from the topological optimization was physically printed on a 3D printer and then used in experimental tests to determine the modal response. Both unoptimized and optimized models were tested both numerically and experimentally for two case studies: fixed model and free model. Finally, the numerical models were validated based on the experimental results.

Chapter 8 presents the overall conclusions of this paper, original contributions and future research directions.

Appendix 1 presents a case study in which the optimization of a communication tower structure is aimed using factorial methods.

For this purpose, a variable geometry model was created that takes into account the specific working parameters, and a finite element model was generated using beam and shell elements for an optimization study. The finite element analysis is automatically updated for each variant of the geometric model. The tower model is subjected to static and dynamic loads, including seismic loads. The optimization solution algorithm used is based on nonlinear programming quadratic Lagrange (NLPQL), which has good accuracy in optimization studies with a single objective function and multiple optimization constraints, such as the present case. Of the three final design variants, the one with the minimum mass is proposed.

The results of this paper demonstrate the effects of different structural component design parameters on both the design characteristics under optimization and the additive manufacturing process and provide essential guidance for the design of these types of structures, thus having wide applicability in almost all technical fields. This is because design optimization helps us to achieve lighter parts, with higher strength in service and at a lower cost.

Chapter 1. Current state of research on applicability of DOE methods in engineering

1.1. Brief history

Design of Experiments (DOE) has undergone four main periods of evolution since its occurrence. Thus, the early period is from 1918-1940 and has its origins in agriculture. Sir Ronald Fisher, working at the time at the Rothamsted Institute of Arable Crops Research in London, and his team discovered and implemented the factorial method (ANOVA), which had a profound impact on the agricultural sciences and was subsequently used with success in other fields.

The second important period is from 1951-1980, which is practically the first DOE industrial era. George Box and K.B. Wilson are noteworthy in this period because they not only improved the factorial method but also laid the foundations for a new method: the response surface methodology. This new methodology has applications in various industries such as automotive, chemical, food, textile, etc.

The third important period is from 1980-1990 and is basically the second DOE industrial era. This period was marked by numerous quality improvement initiatives in many companies. Of note during this period is Dr. Genichi Taguchi who was an engineer and statistician at Nippon Telephones & Telegraph in Japan. At the beginning of 1950s, Taguchi developed a methodology for applying statistics to the production process to improve the quality of manufactured products, with the ultimate goal of having the most robust product possible.

The modern era begins with the 1990s and is basically the fourth important period in the history of DOE methods. It is marked by the widespread use of computers, the extensive use of DOE methods in Six-Sigma and in business, and also the combination of DOE methods with various numerical methods in computer experiments.

1.2. Knowledge required to apply DOE in industry

In order to carry out a successful industrial experiment, the following skills are generally required [1]:

- Planning skills – Understanding the importance of the experiment to a particular problem, time and budget needed for the experiment, how many people are involved in the experiment, determining what needs to be done, etc.

- Statistical skills – Involves performing a statistical analysis of the data obtained from the experiment, assigning variables and interactions to different columns of the design array (experimental design), interpreting the results of the experiment to make sound and valid decisions for possible improvement, etc.

- Teamwork skills – Involves understanding the objectives of the experiment and the common understanding (by all team members) of the experimental objectives to be achieved, better communication between people with different skills and learning from each other, etc.

- Engineering knowledge – Determining the number of levels (possible values) of each input variable, the range of variation of each variable, determining what is measured in the experiment, determining the capability of the measurement system, determining which variables can and cannot be controlled during the experiment, etc.

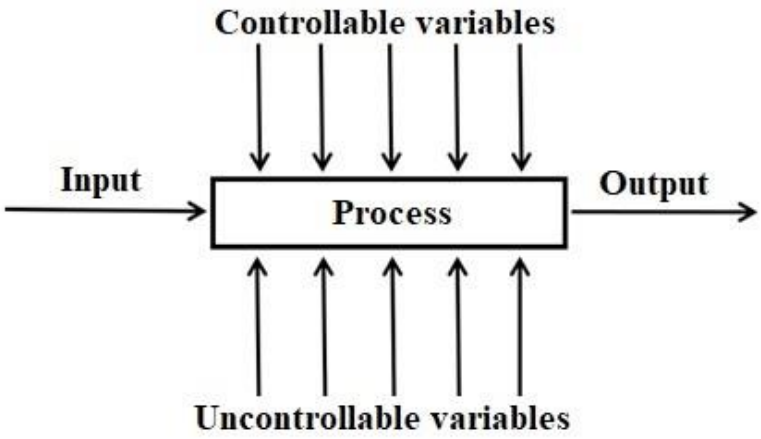


Fig. 1.1. The general design of a process/system [1]

1.3. General procedure

The following sequence of experimental stages can be adopted [1]:

1. Identify the process to be studied and the purpose of conducting the experiments
2. Identify the outcomes, also called response variables, that need to be improved
3. Determine the accuracy of measurements based on repeatability and reproducibility studies
4. Identify the input variables that can be controlled, also called factors
5. Choose the limits or levels for each factor. Usually, two levels will be used for each variable. Mark the upper level with “+” and the lower level with “-“.
6. Establish and document the design of the experiments. This includes:
   - all the different combinations between levels illustrating what the levels of the variables will be for different combinations
   - how many times each combination will be experimented (this action is called replication)
   - the sequence of all trials, chosen in a random order (called randomization)
7. The experiment is carried out, strictly following its design.
8. The data obtained are analyzed and conclusions are drawn. Specialized software is available to carry out the calculations.
9. If the conclusions lead to changes to improve the process, verify the results and authorize the new process.
10. Determine whether there are additional experiments that can be performed. These are planned and carried out starting again with step 5.

1.4. DOE – Basic principles

A better understanding of the problem

Research has shown that one of the key reasons why an industrial experiment is unsuccessful is due to a lack of understanding of the problem itself. The success of any industrial experiment depends to a large extent on the nature of the problem, the professional training and the hard work of the team.

Setting the working strategy

The successful implementation of DOE requires a mix of statistical, planning, engineering, communication and not least teamwork skills. Establishing the working strategy should be treated as an integral part of designing effective experiments [2].

Selecting the appropriate response or quality feature

A response is the performance characteristic of a product that is the most important cue for customers and often reflects the quality of the product. It is important to choose and measure an appropriate response for the experiment. The following ideas may be useful to those involved in selecting the quality characteristics for industrial experiments [2]:

- Use responses that can be accurately measured
- Use responses directly related to the energy transfer associated with the fundamental mechanism of the product or process
- Use responses that are complete, i.e., those responses that perfectly cover the input-output relationship for a given product or process

Selection of process variables

The selection of process variables is useful for reducing the number of process variables to a manageable number which implicitly leads to a reduction in the number of experimental trials and in the costs associated with the whole experimental process. For example, seven input variables can be used while performing only eight experimental trials. It is advisable not to invest more than 25% of the experimental budget in the first stage of any experiment. Once the key variables have been identified, their interactions can be studied using full or fractional factorial experiments [3].

Using the grouping strategy to increase the efficiency of the experiment

The grouping strategy can be used to minimize the influence of various external factors, such as: change of working machine, change of team members, location of the experiment or the actual day of the experiment, etc.

Conduct experimental trials to confirm the response

It is necessary to repeat confirmatory trials (samples) to verify the results of the statistical analysis. Some of the possible causes for not achieving the objective of the experiment are wrong choice of design for the experiment, failure to identify the key process variables affecting the response, or inadequate measurement system to perform the measurements [3].

1.5. Conclusions

In general, when using DOE methods, the aim is to determine the effect of each input variable on the response and how the input variables may interact with each other. DOE allows the development of a mathematical model that predicts how input variables interact to create output variables (responses) in a process or system. It is also very important to determine the values of the input variables that optimize the responses – for example maximizing one variable but minimizing another.

Thus, DOE aims to obtain and analyze the quantitative experimental data required to determine the relationship between the factors that influence a process and the outcomes of that process, through an approach that applies statistical principles and techniques that ensure that valid and consistent conclusions are generated and supported based on the minimum amount of data required, which involves minimal consumption of resources (money, time, etc.). The data collected usually requires the use of statistical analysis tools for analysis and interpretation (ANOVA, correlation analysis, regression analysis), which makes it useful to purchase special calculation programs.

In the case of certain methods, such as the Taguchi method, in addition to those mentioned above, it is very important to determine the input variables needed to minimize both the variability of the response and the effect of uncontrolled or difficult to control input variables.

DOE is a methodology for the systematic application of statistics in experimentation and can be used for a wide range of experiments for various purposes in almost all fields of engineering and even in marketing or business.

Chapter 2. Factorial Methods

2.1. Factorial designs

Experimental designs are the strategy for scheduling (planning) experimental trials to obtain useful results with a satisfactory level of confidence. An experiment is factorial if each level of a factor is combined with each level of the other factors in the experiment.

Factors are defined as parameters that characterize inputs to the system under research, also called input factors or input variables. The parameters obtained from the experiment are called results, outcome factors, response variables or output variables, i.e., the results recorded in the experiment [4].

The factors (variables) of an experiment can be:

- quantitative factors, i.e., measurable, e.g. concentration, temperature, pressure, humidity, etc.;
- qualitative factors are non-measurable factors that can be identified by an attribute, e.g. type of raw material, type of catalyst, work shifts in a factory, etc.

Qualitative, non-measurable or attributive factors can only be used in array models.

In factorial experiments, the quantitative input variables (controlled factors) are forcibly modified at various discrete subdivisions called levels, the experiment being carried out according to a schedule (design) so chosen that with a minimum of measurements and processing calculations of the experimental data, a model of the process (phenomenon) under research is obtained with acceptable accuracy [5]. The term “independent variable” is not recommended to be used (according to ISO 3543-3:2013). The first to propose such programs were Box and Wilson [5].

Experimental designs can be of the form:

- 2-level *full factorial design*, also known as "2^k" design or a three-level factorial design or a "3^k" design; “*k”* is the number of factors. By levels we mean the number of different values that a variable/factor can take, according to its discretization; for each factor the same number of levels is chosen, although there are methods of designing experiments that allow differentiation of the number of levels for variables [6];
- *Fractional factorial design* results by dividing a "2^3" design into two "2^2" subsets; the fractional experiment may be one half or one quarter of the full factorial experiment;
- *Taguchi experiment designs*, which belong to the category of fractional designs;
- *Latin square and Greek square* experimental designs.

2.1.1. Randomized complete block experiment

The term *block* originated in experiments in agriculture, where a land was subdivided into sections that had common conditions, for example, the same exposure to wind, proximity to groundwater or thickness of topsoil. In other situations, blocks are based on common raw material/raw material lots, same operators, number of units studied on the same day, products produced on the same shift, etc. Therefore, a factorial experiment is conducted “in blocks” if homogeneous (common) conditions are maintained during the experiments. The Randomized Complete Block Design (RCBD) is a method of designing experiments based on blocking.

Blocking means an arrangement of experimental units in blocks. In an experiment, there are always different factors that can influence the outcome (response variable). Some of these factors are uncontrollable (random), therefore they have to be randomized (randomly allocated) during the conduct of the experiment so that, on average, their influence will be negligible [7].

2.1.2. Full factorial design

The full experimental design is a common and intuitive strategy for designing experiments. This design is used in optimization studies. An experiment is full factorial if each level of a factor is combined with each level of the other factors in the experiment, i.e., the experimental samples are given by any possible combination of factor levels. As mentioned before, factors can be quantitative, e.g. temperature and pressure, or they can be qualitative, e.g. two machine tools or two operators (k = 2) [8].

In the context of the problem of factorial experiments, the notion of *degree of freedom* is also used. Mathematically, the number of degrees of freedom associated with a factor is equal to the number of levels assigned to this factor, minus one. In the case of an interaction between two factors, the number of its degrees of freedom is calculated as the product of the numbers of the degrees of freedom of its component factors. Sometimes in the literature on the design of experiments, full factorial experiments are encountered where the center point (or base level) of the factor space is also added in the selections [8].

2.1.3. Fractional factorial design

When the number of factors increases, the full factorial design can become very difficult to perform because of the large number of experiments. The idea of designing a fractional factorial experiment is to run only part (a subgroup or fraction) of the full factorial design. The fractional factorial design can be one half of the full experiment when the full design/experiment is decomposed into two subsets, or it can be one quarter of the full experiment, etc. For example, a 2^3 full experimental design is decomposed into two 2^2 subsets. If there are only three factors of the experiment, then the full factorial experiment requires eight runs of the experiment, and the split-half experiment will require four runs. As the number of factors increases, the fractions become smaller: fractions 1/8, 1/16, etc. The construction of the two-level fractional factorial experiments is based on the structure of the two-level full factorial experiment. Generalizing from a full 2^k experiment, the fractional half experiment is 2^{k-1} [3].

2.1.4. Central composite design

The central composite design is a 2^k full factorial experiment in which additional experimental points are added to the 2^k experimental points: the center point of the experiment and $2k$ “star points” at α distance (sometimes written “alpha” distance) from the center point, resulting in a selection size of $2^k + 2k + 1$. Since the experimental points are symmetric with respect to the center point of the experiment, it is referred to as the “central”. There are situations where two or more experiments are needed at the center point and situations where one or two experiments at the center are sufficient.
By noting with n_c the number of experiments in the center, the total number of experimental points N for k factors will be: $N = 2^k + 2k + n_c$. Center point experiments provide information about the existence of response surfaces with curvature [8].
If a curvature is found in the system, the addition of “star points” allows efficient of the purely quadratic terms. The central composite design is defined according to the criterion on the basis of which the distance α is calculated, the most common in practice being orthogonal second-order and rotatable second-order central composite experiments. The fact that there are more selections than strictly necessary for a bilinear (2^k) interpolation allows to estimate the curvature of the experimental space.

2.2. Taguchi method

The Taguchi method was initially criticized by some classical Western statisticians but others accepted it, and many of the concepts he introduced were later used successfully. Thus, Taguchi has been identified with the emergence of what has come to be called today *quality engineering*. The current aim of *quality engineering* is to move quality improvement actions upstream from the production stage to the product design stage.

As his method itself demonstrates, Taguchi's main concern is the deviation of an output variable from its nominal value. Uncontrolled parameters ("noise parameters") are often responsible for this deviation and therefore Taguchi's approach in experimental design aims to design products or production processes that are resilient (robust) to these types of parameters (temperature, humidity, run time, machine tool inaccuracy, human error, etc.) [9]. Special attention is paid to the design of systems so that their performance is not sensitive to environmental changes. The effect of different input parameters (design or "noise") on the performance characteristics in a condensed set of experiments can be examined using the orthogonal array models proposed by Taguchi.

2.2.1. Robust design

Robust design is an experimental method based on statistical principles to produce quality products whose performance is insensitive to variation in "noise" parameters. Classical DOE has focused on how different design parameters influence the average level of performance.

In Taguchi's DOE method, which belongs to the category of "robust design" methods, the variation of results is more interesting to study than their average. Taguchi's view is that minimizing the variance of results is the main means of quality improvement [10].

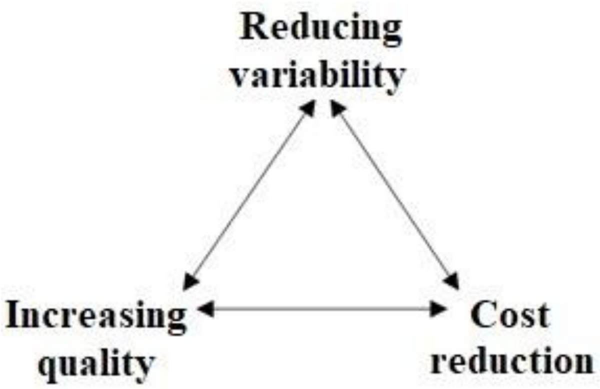


Fig. 2.1. Interdependence of variation in results – quality – costs

2.2.2. Orthogonal arrays

Taguchi designed a series of orthogonal arrays to be used in the experiments. These ma arrays trices are actually models of fractional factorial designs.

He uses two arrays to perform the experiment:

- The inner array is used to study the effects of the design parameters we want to study
- The outer array is used to model uncontrolled or difficult to control parameters that may affect process performance or product quality

One of Taguchi's simplest orthogonal arrays is the one in Table 2.1:

Table 2.1. $L_8(2^7)$ Orthogonal array

Exp.	P1	P2	P3	P4	P5	P6	P7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

2.3. Response surface methodology

The response surface methodology (RSM) is based solely on numerical simulation and is suitable for case studies using up to about ten input parameters. Response surfaces provide approximate values of the output parameters from the whole optimization domain under consideration, without the need to perform a full solution. For this reason, the optimal results are approximate and must be verified by specific methods. The response surface methodology also has advantages because it "covers" the whole optimization domain (Fig. 2.2), the computation time is reduced and the modification of the optimization criterion and the rerun of the solution are done quickly and easily [11].

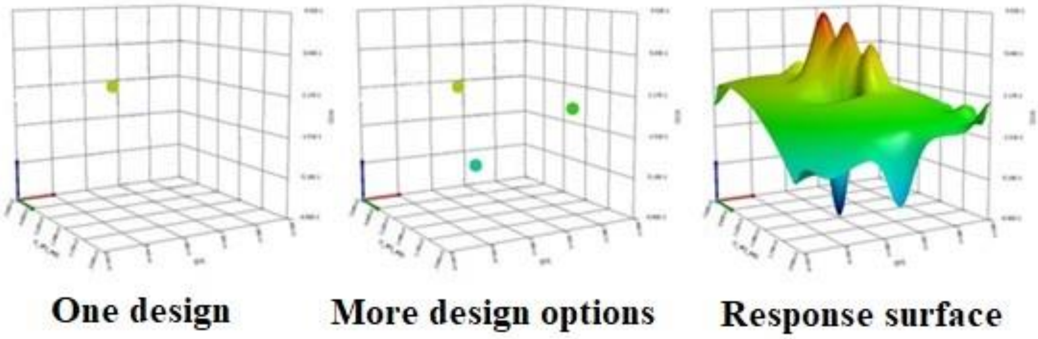


Fig. 2.2. Graphical representation of response surface [12]

2.3.1. Types of response surfaces [12]

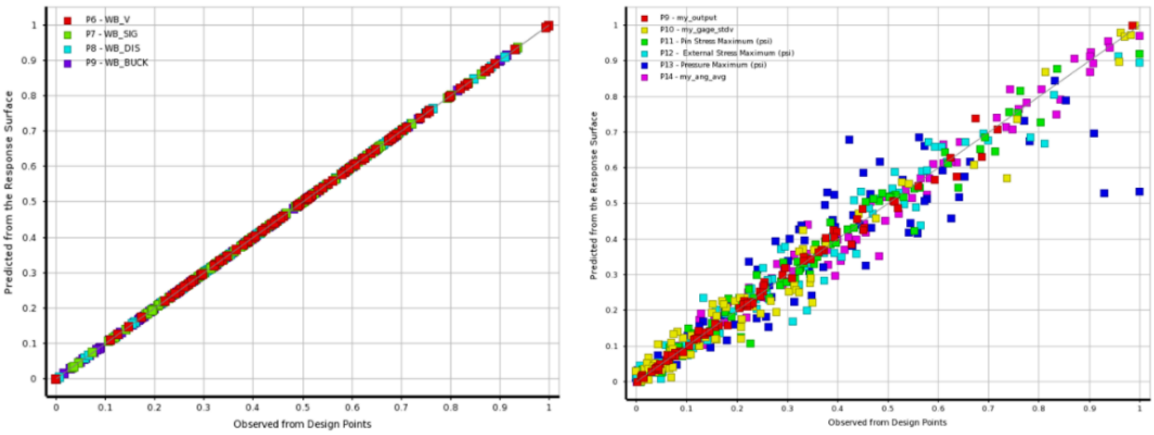
We can illustrate some mathematical models:

- **Standard Response Surface**
  It is effective when the variation of the output parameters is progressive compared to the input parameters.
- **Kriging**
  It is effective in a study using a large number of design variants and is suitable for responses involving nonlinearities; it always uses checkpoints to control the accuracy of responses.
- **Non-parametric Regression**
  It is suitable for answers involving nonlinearities but has the disadvantage of a slow outcome.

- **Neural Network**
  It is suitable for answers involving large nonlinearities but control over the algorithm is very limited.
- **Sparse Grid**
  It is suitable for studies that contain discontinuities and the outcome is usually achieved quickly. A good choice for many case studies is the Kriging mathematical model with active refinement.

2.3.2. Checking the accuracy of the outcome

A quick method of assessing the quality of the response surface is the "Goodness of fit" (GoF). In essence, the GoF usually summarizes the discrepancy between the observed values and the expected values according to the model in question. Such measures can be used in statistical hypothesis testing, for example, to test whether two samples are drawn from identical distributions or whether the frequencies of the outcome follow a specified distribution. Fig. 2.3 shows the assessment of the quality of the response surface using GoF. The values plotted on the abscissa result from the solution of the design variants and the values plotted on the ordinate are determined from the solution of the response surface. In Fig. 2.3 (a) it can be seen that the working parameters are perfectly aligned with the graph which means that the resulting response surface is excellent and in Fig. 2.3 (b) they are rather scattered which means that the response surface is of average or even poor quality.



(a) (b)

Fig. 2.3. Rapid assessment of the response surface quality: excellent quality (a), average/poor quality (b) [12]

2.3.3. Processing the results

The graphical representation of both 2D and 3D response surfaces is useful for establishing desired response values and operating conditions. In a 2D graphical representation, the response surface is viewed as a two-dimensional plane in which all points having the same response are connected to produce constant response contour lines. A 3D graphical representation generally displays a view that can provide a clearer picture of the response. If the model under study does not contain interaction effects, the predicted response surface will be a plane (i.e., the contour lines will be straight), whereas if the model contains interaction effects, the contour lines will no longer be straight but curved [12].

2.4. Conclusions

In experimental research, the term control variable is used to designate the variable whose effect we want to control or eliminate. This checks the certainty of the relationship between the independent variable and the dependent variable, or, in other words, that the effect obtained is not explained by the presence of a variable other than the independent variable. Basically, if the influence of other variables is controlled or eliminated and the value of the dependent variable is maintained, then the cause-effect relationship (independent variable-dependent variable) is a true one. In general, we aim to control those variables that are suspected of influencing the research results.

The experimental design reflects the general structure of an experiment (variables, experimental conditions) and does not give details of how it is conducted, assumptions, etc. There are several elements involved in classifying experimental designs: the number of independent variables, the number of experimental conditions given by the levels of independent variables, and the use of a single category of study designs (comparisons within the same category) or several categories (comparisons between them). In its simplest form, the experiment involves two measurements: one before the experimental manipulation and one after. The difference between the first measurement (pre-test) and the second measurement (post-test) reflects the effect of the independent variable.
Unlike scientific research, where full factorial experiments are conducted, the design of experiments (DOE) proposed by Taguchi is simplified and based on fractional factorial experiments, which allows experimental data to be collected by changing only some of the variables and selecting a few relevant levels of their values (minimum two levels, usually three levels are preferred and rarely four levels are considered. Three, four or more levels are only used for continuous variables that influence each other.

Chapter 3. Optimization process in mechanical engineering

3.1. Concepts and structure of the optimization process

Optimization is essentially a scientific option and consists of systematically developing and sorting possible solutions to an engineering problem. The final aim of optimization is to select the solution which, within a reference framework defined by the conditions accepted or imposed initially, leads to the most advantageous use of the resources available for its realization. Optimization of a machine, plant or construction of a given type can be done by optimizing its separate components, sub-assemblies or distinct structural parts, the structural frame being one of them. This will be referred to in particular below. The mathematical foundations of the optimization processes are operational research, linear programming, dynamic programming, geometric programming, genetic algorithms, etc.

The main purpose of optimizing a structure – or, in other words, designing the structure optimally – is to determine its shape. The determination of stresses and displacements is a further step in the design process, where it is checked whether the shape and dimensions of the structure meet the requirements of the intended purpose [13].

The most commonly used criteria underlying the design models for optimizing structures are: minimum weight, minimum stresses (maximum strength), minimum potential deformation energy, maximum stiffness, minimum displacements, maximum stiffness for a given weight, shape of equal strength, minimum cost, etc.

From a mathematical point of view, it is not a matter of solving a system of algebraic, compatible equations that has a unique solution. The mathematical algorithm of the optimization process is usually a “heuristic strategy” to find the best solution out of the set of possible ones.

3.2. Use of optimization in engineering

The optimization process is a component of the design and manufacture of a product, but in the end the optimized structure must also meet other conditions or constraints, which are always present in mechanical engineering, i.e., “the theoretical” outcome of the optimization process must be validated, in the end, by technological, assembly, transport, operating, aesthetic, ergonomic, ecological, etc. considerations. These constraints are formulated mathematically in the form of relationships between design variables. They limit the range of variation of these variables and thus “the design space” in which the optimal solution is sought [13].

Technological constraints. Any structure is built within a set of technological conditions that exist or are accessible to the designer, which determine some “adaptations” of the product, since each type of technological process has its advantages, limitations and disadvantages.

Assembly conditions. The structure cannot be accepted for execution if it does not meet the assembly conditions. All components and sub-assemblies of the structure must be capable of being individually manufactured and then assembled under the conditions of accuracy, sealing, etc., laid down in the design.

Transport conditions. The structure as a whole or its components – if the structure is of large dimensions – must be transported to the beneficiary under conditions which do not affect the geometrical shape, dimensional accuracy or functional parameters of the product. In certain situations, these considerations may have a decisive influence on the configuration of the structure, the construction or technological solutions to be used by the designer.

Operating conditions. The final validation of any engineering activity is the in-service behavior of the product, machine, device or installation that was the objective of the designers, executors, users, etc.

3.3. Structural optimization

Structural optimization involves determining the design variables that control the shape, material properties or dimensions of a structure so as to meet certain constraints and improve certain properties to achieve optimal structures [14].

In the process of designing structures, in various engineering fields, designers choose the best decision option at each step related to structural and non-structural aspects such as stiffness, strength, aesthetic properties. In other words, they make decisions to achieve the best design, so the structural design process can be seen as the optimal design even if it does not expressly aim to find an optimum. Structural optimization is regarded as the application of optimization methods in structural design [15].

Mechanical properties including node displacements, stresses in elements, vibration frequencies, buckling loads are taken as design variables. The structural optimization problem can be formulated, as an alternative, to pursue the maximization of a mechanical property, subject to cost constraints. Although there are multiple formulations of the optimization problem, e.g. design for minimum weight, design for maximum stiffness, the term structural optimization or optimal design refers to all types of optimization problems associated with structural design.

The finite element method can be used as the numerical core for general problem solving for the most diverse types of structures and stresses. There are a multitude of calculation programs that use FEM, which provide all the necessary data to be processed in the optimization algorithm.

In order to work in an integrated mode, it is necessary to use “software” automatism to define the problem and solve it automatically using FEM. It is also necessary that the software can automatically retrieve the data from the finite element program and use it further. The formulation of the optimization problem should be done automatically. Since solving problems using FEM is a computationally expensive process, it is necessary to minimize the number of model runs using FEM [16]. In order to avoid damage to the modelled structural model as a result of geometry or topology adjustment in the optimization process, it is necessary to define a set of constraints in addition to the constraints related to the structure configuration (stresses, strains, displacements). For a good positioning of the search process in the solution space, it is very useful to use the sensitivity analysis. Using the sensitivity analysis, the search space is reduced to the template suggested by the sensitivity coefficients [12].

In order to increase the efficiency of the method a dual strategy is used, in which optimization with discrete variables is performed after optimization with continuous variables. Statistically it has been established that this dual method is at least one order of magnitude more efficient than other methods for optimization problems with more than 20 variables.

A novel element is the possibility of using a discrete set for the design variables, which represents a pragmatic approach to the structural optimization process by being able to obtain values that have practical applicability [17]. Classical algorithms have as a connecting element the use of the finite element method as a calculation procedure for the stresses and strains of the analyzed structure.

The analysis of optimization methods, both in terms of the use of FEM for the calculation of displacements and strains and in terms of ease of implementation, as well as the need for a certain "hardware" and "software" accessibility, leads to the conclusion that two options can be considered for structural optimization: either the use of an optimization module coupled with a structural analysis module with FEM or the creation of its own optimization module, coupled with a structural analysis module with FEM, which meets certain specific requirements [18, 19, 20]

3.4. Use of structural analyses in the optimization process

3.4.1. Definitions and basic terminology

Linear static analysis

For a linear static analysis, the displacements $\{u\}$ are solved using the matrix equation below:

$$[K]\{u\} = \{F\} \tag{3.1}$$

Hypothesis:

- The matrix $[K]$ is constant
  - We assume a linear elastic material behavior
  - We use the small-deformation theory
  - Some nonlinear boundary conditions may be included
- Force $\{F\}$ is statically applied
  - Time-varying forces are not taken into account
  - No inertial effects included

Dynamic analyses

For dynamic structural analyses we have the following general equation which, depending on the type of analysis, takes a specific form [21, 22, 23]:

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{F\} \tag{3.2}$$

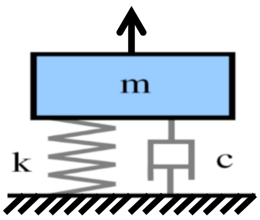


Fig. 3.1. Damped mechanical structure [22]

Where:

- $[M]$: mass matrix
- $[C]$: amortization matrix
- $[K]$: stiffness matrix
- $\{F\}$: force vector
- $\{\ddot{u}\}$: acceleration vector
- $\{\dot{u}\}$: velocity vector
- $\{u\}$: displacement vector

For linear static analysis, Young's modulus and Poisson's ratio are necessary and sufficient. If we want to apply inertial or gravitational loads in a static structural analysis it will be necessary to define the density [24].

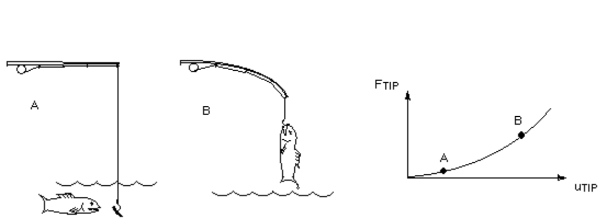
For linear dynamic analyses that are defined as a function of frequency or time, Young's modulus, Poisson's ratio and density are necessary and sufficient [25].

3.4.2. Definition of material properties

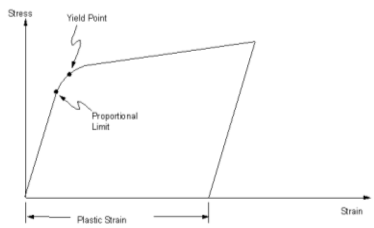
For nonlinear static and dynamic structural analysis, in addition to the above-mentioned properties, it is also necessary to add those defining the nonlinear behavior of the material (e.g. stress-strain curve).

There are three main categories of nonlinearities [22, 26, 27]:

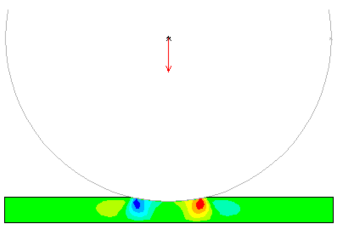
- *Geometric nonlinearities*: If a structure has large deformations, changing the geometrical configuration can cause nonlinear behavior (Fig. 3.2 (a)).
- *Material nonlinearities*: A nonlinear stress-strain relationship, such as the plasticity shown in Fig. 3.2 (b), is another example of nonlinearity.
- *Contact nonlinearities*: The inclusion of contact effects is a type of nonlinearity due to "status" change, since a sudden change in stiffness can occur when the bodies involved move in and out of contact (Fig. 3.2 (c)).



(a)



(b)



(c)

Fig. 3.2. Types of nonlinearities: geometric (a), material (b), contact (c) [22]

In principle, in dynamic analyses it seems convenient to use the same finite element model as the one built for static analysis. However often the static model contains more details than are needed in the dynamic analysis, so dynamic condensation or sub structuring is used to reduce the order of the dynamic problem before solution [28].

3.5. Numerical correlation of parameters in the optimization process

In a DOE study, the number of design variants increases rapidly as the number of input parameters increases, which can reduce the efficiency of the study. It is recommended to exclude unimportant input parameters to reduce unnecessary sampling.

Parameter correlation and monitoring allows us to identify important parameters and the correlation matrix, its graphical representation and sensitivity plots also help in understanding the parametric relationships [29]. Parameter correlation performs simulations based on random sampling of the optimization domain so that it is performed between all parameters.

The number of samples required to compute the correlation is determined based on the convergences of the standard deviations or means of the output parameters. At each iteration, the convergences of the standard or mean deviations are checked against the precision initially imposed by the engineer. The iteration stops when the accuracy is met or the number of samples exceeds the limit initially set, whichever comes first.

3.5.1. Correlation methods

Two of the most commonly used correlation methods are:

1) Linear correlation (Pearson method)

- Uses real data for correlation assessment
- Detects a linear relationship between two variables

2) Rank correlation (Spearman method)

- Detects a monotonic relationship between two variables
- It is less restrictive than a linear relationship
- Is considered to be more accurate and is therefore recommended

The Pearson and Spearman methods are based on the same equation. The difference is that the first one uses the actual values and the second one ranks (steps) [12].

$$r = \frac{\sum((X-X_M)(Y-Y_M))}{\sqrt{\sum(X-X_M)^2 \sum(Y-Y_M)^2}} \tag{3.3}$$

Correlation matrix

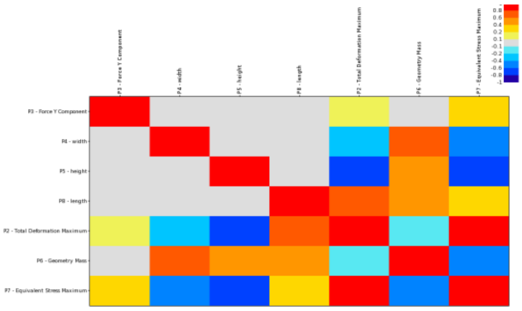
The correlation coefficient indicates whether there is a relationship between two variables and this relationship is represented by a positive or negative number. The correlation matrix allows us to analyze the relationship between any two variables.

The closer the value is to the extremes -1 or 1 the stronger the relationship is. In Fig. 3.3 (a) we can see the values and in Fig. 3.3 (b) the colored rectangular areas corresponding to these values.

The red areas on the diagonal of the matrix and the dark blue areas correspond to the values 1 and -1 respectively. The positive values indicate that the two variables are directly proportional and the negative values indicate that the two variables are inversely proportional. The grey areas correspond to small values and this indicates that there is a weak relationship between the two variables, so they influence each other very little or not at all [12].

Name	P3 - Force Y Com... (N)	P4 - width	P5 - height	P8 - length	P2 - Total Defo... Maxim... (mm)	P6 - Geom... Mass (kg)	P7 - Equiva... Stress Maximum (MPa)
Correlation Matrix (Linear)							
P3 - Force Y Component	1	0.074527	0.059844	-0.073192	0.11057	0.032703	0.31346
P4 - width	0.074527	1	0.042937	-0.042937	-0.27386	0.62981	-0.45095
P5 - height	0.059844	0.042937	1	-0.033593	-0.66274	0.50434	-0.71746
P8 - length	-0.073192	-0.042937	-0.033593	1	0.68988	0.51813	0.36552
P2 - Total Deformation Maximum	0.11057	-0.27386	-0.66274	0.68988	1	-0.12125	0.88254
P6 - Geometry Mass	0.032703	0.62981	0.50434	0.51813	-0.12125	1	-0.43582
P7 - Equivalent Stress Maximum	0.31346	-0.45095	-0.71746	0.36552	0.88254	-0.43582	1

(a)



(b)

Fig. 3.3. Correlation matrix: specific values (a), corresponding areas (b) [12]

3.5.2. Sensitivity chart

Sensitivity shows how different values of an independent variable will influence a particular dependent variable. It is "positive" when increasing the value of an input parameter leads to an increase in the value of an output parameter. Sensitivity is also "negative" when increasing the value of an input parameter leads to a decrease in the value of an output parameter. Statistical sensitivities are based on Spearman rank correlation coefficients that take into account both aspects at the same time.

3.6. Conclusions

Optimization based on numerical methods is done through an iterative process, defining an initial state used as a starting point for a systematic search to improve the structure. The iterative process is stopped when all criteria are satisfied, so that the current configuration obtained is as close as possible to the optimum sought.

The optimum design must be in the feasible range. It can be said that the optimum can be found by solving differential equations with a clearly specified form. This observation may lead us to the partly true conclusion that the optimum could always be found with the help of a clearly formulated algorithm. However, it should be noted that the equations defining the optimality conditions are subject to extremely restrictive conditions.

The domain of analysis represents a model of the body under study or only of a significant region belonging to it. It is obtained through a process of idealization of the original body geometry and sometimes selection of the region of analysis. Idealization is necessary to reduce the complexity of the geometric configuration of the original body, to reduce the effort of preparing the input and computation data.

Lately, finite element method based analyses have addressed advanced optimization problems and a number of material constitutive laws have been implemented to allow the computation of non-classical metallic materials (composites, plastics, etc.) in both linear and nonlinear domains. New finite elements have been developed, much more powerful than those used initially, there are many types of finite elements of effect coupling (the thermal effect with mechanical, electrical or magnetic one, etc.). To facilitate the discretization operation, new automatic discretization techniques have been developed, based on the computational error obtained in a first gross discretization.

The progress and success of the finite element method in recent times is due to the spectacular development of numerical computers as well as finite element calculation programs. It can be firmly stated that nowadays design is unthinkable without the use of the finite element method in calculations of strength, optimization, stability, fatigue, etc.

Chapter 4. Topology optimization

4.1. Introduction

The term *topology* comes from the combination of the Greek nouns *topos* (τόπος) and *logos* (λόγος), meaning place, respectively study. So, topology literally means “the study of place”. No overriding assumptions about form are necessary.

Topology differs from Euclidean geometry in the way it considers the equivalence between objects. In Euclidean geometry, two objects are equivalent if they can be transformed into each other by isometries – transformations that preserve the value of angles, lengths, areas and volumes. Topology is a branch of mathematics, specifically an extension of geometry that studies deformations of space by continuous transformations.

Topology optimization of a structure consists of finding the best shape for that structure that meets the required strength criteria in service, connectivity, number and location of holes, etc., so that one or more of the objective functions initially set are minimized or maximized. This optimization method is based exclusively on numerical simulation.

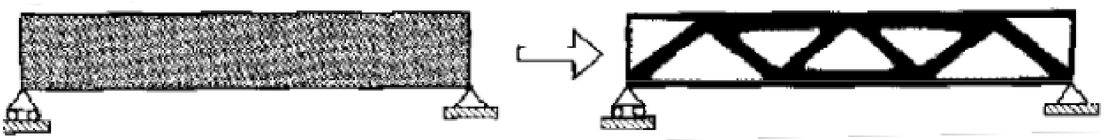


Fig. 4.1. Topology optimized model [30]

4.2. Integrated structural optimization techniques

The aim of implementing integrated topology optimization techniques in computer-aided engineering programs is to automate design procedures and decrease design time. The core of the design process is the software used, which also has a calculation module based on the finite element method. In this system topology optimization procedures must be introduced in the initial stages [31].

Current structural optimization techniques are summarized in Fig. 4.2. Topology optimizations aim at the optimal distribution of material in a structure for the given stresses. It is a multi-criteria optimization, the procedure finding solutions for multiple stresses. Topology optimization is also called footprint optimization. It is applied to thin-walled structures. Shape optimizations are local optimizations and aim to find the optimal shape for a single structure’s stress. It is accessed when elaborating details. Parameter optimizations aim at finding the parameter value that best satisfies user-defined criteria and allows minimization of an objective function [32].

Topology optimization is the determination of the general spatial configuration of the component elements or links of a structure. The aim is the most rational distribution of all the material used. Topology optimization is also called generalized shape optimization.

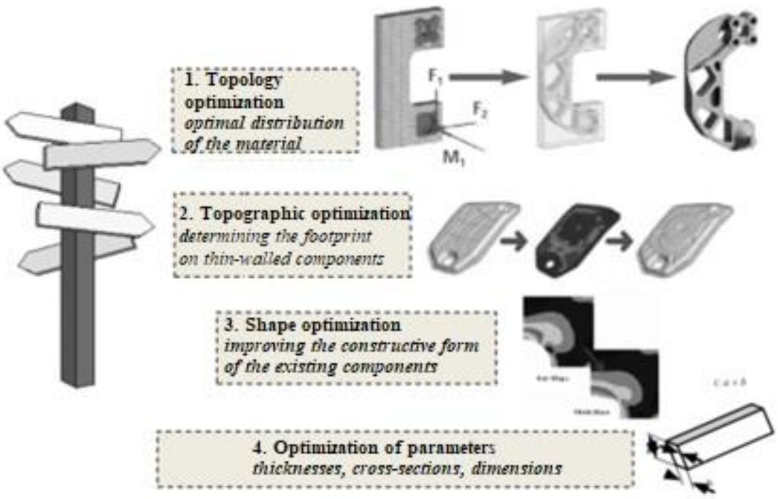


Fig. 4.2. Current techniques of optimization [33]

4.3. Classical approaches

- Dimensioning of trussed structures

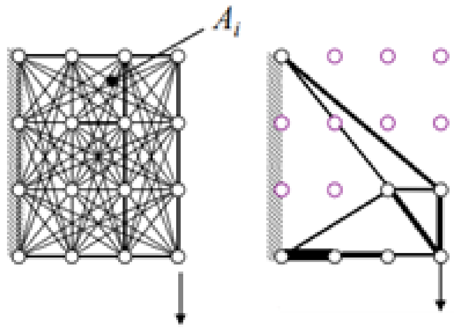


Fig. 4.3. Trussed structure [34]

- Optimization of plate thickness

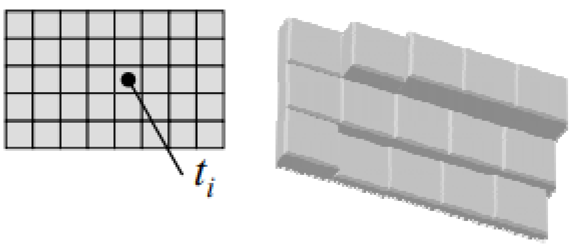


Fig. 4.4. Variable thickness plate [34]

- Homogenization method

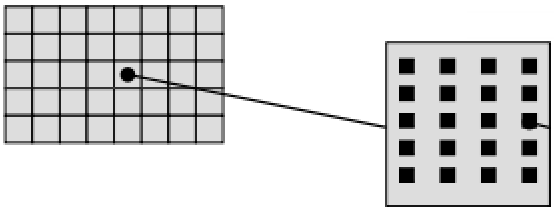


Fig. 4.5. Microstructure sizing [34]

4.4. Methods for setting up topology optimization problems

The most commonly used mathematical methods in the topology optimization process are those based on the material distribution mode:

- Penalty method
- Homogenization method

The basic idea is to geometrically represent the structure as a black and white rendering of the image of the model undergoing the topology optimization process [34].

We will determine the points in the reference space which must be:

- Material areas (χ=1)
- Vacuum areas (no material) (χ=0)

So that the criteria set for the objective functions are met.

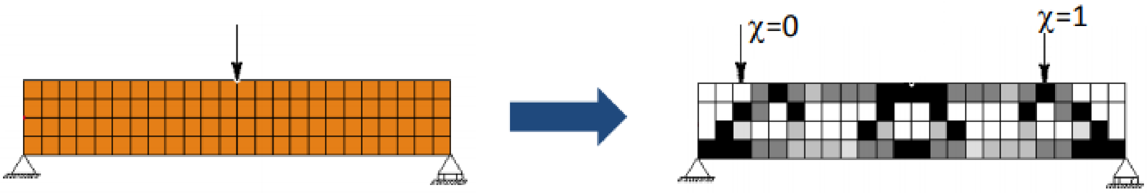


Fig. 4.6. Geometric representation of the structure [34]

In the topology optimization process we have to be careful about the occurrence of possible numerical instabilities [35]. These can manifest themselves as follows:

- “Chessboard”: Formation of regions with alternating solid and vacuum elements arranged like a chessboard.

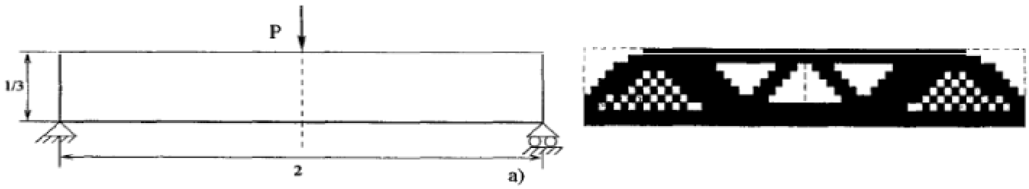


Fig. 4.7. Regions with alternating solid and vacuum elements [36]

- Discretization network dependence: qualitatively different solutions for different element sizes or types of discretization.

  Solution for:

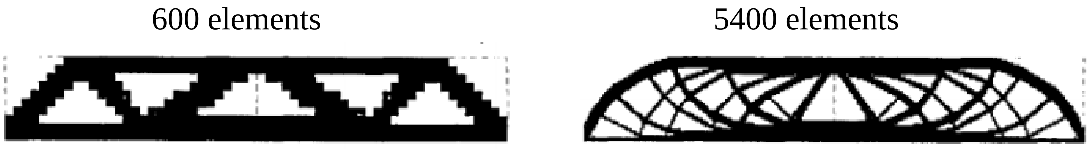


Fig. 4.8. Discretization network dependence [36]

4.5. Practical considerations

By performing a topology optimization analysis, a structurally optimal geometry design is determined for a well-established area of the model with specific design goals and constraints [37].

Topology optimization is an optimization that is based on a set of loads and boundary conditions provided by one or more analyses that precede it. The topology optimization analysis is preceded by a structural analysis or a combination of structural analyses coupled together. The loads and boundary conditions defined in the upstream analysis are used to create an optimized structural component based on the objectives and constraints specified in the topology optimization analysis. It is necessary to use the same boundary conditions for all previous analyses [38].

Once the topology optimization analysis is completed, the new design is validated. It is important to note that in order to perform a validation, the topological optimization analysis must be solved beforehand.

Validation of the new design becomes possible by converting it from STL format to a solid geometric model which is then analyzed using the same loads and boundary conditions used in the original structural analysis. After validation, the new design is proposed for manufacturing.

4.6. Conclusions

Today, the optimization process is part of the design and manufacturing process. The final model resulting from the iterative optimization process must satisfy the conditions and constraints imposed on the operation of the final product. Of the existing methods of optimizing structures that are used to date, topology optimization is one of the most widely used. By using topology optimization one can reduce the material volume and increase the stiffness of the optimized model.

Topology optimization of mechanical structures is a process of determining the most efficient geometrical shape of the part. The optimized shape of the part results from the application of optimization criteria. Structural optimization is intended to determine minimum stresses in the part, high stiffness, minimum displacements, minimum cost and minimum part mass.

Optimization is a complex, iterative mathematical process that requires the use of a computer and dedicated software. In general, any detail of a part can be optimized, such as: the gauge dimensions of the part, the shape of the part by creating connection radius, natural frequencies, etc. From the initial stages of the design, the design of the structure is oriented towards the actual type of stress.

Chapter 5. Improving the modal response of a mounting bracket using lattice optimization

5.1. Introduction

Additive manufacturing (AM) enables design features that are impractical or impossible to fabricate using conventional manufacturing methods. One class of newly enabled features is that of internal lattice structures. Due to their high stiffness to weight ratio and tunable properties, lattice structures, alternatively known as cellular structures, have been extensively connected to metal additive manufacturing [39, 40] and gradually applied to various fields, such as aerospace [41] or biomechanical [40, 42]. The development of 3D printing technologies made nowadays possible to produce such metallic and non-metallic materials. Classically, lattice structures can be divided to be periodic or stochastic from their layout patterns. Extensive studies have been carried out to demonstrate the superior performance of periodic lattices compared to stochastic ones. In a review on design and structural optimization in AM [43] it was emphasized that expertise-driven structural optimization such as latticing is not always having lightweight as an objective, as stiffness is greatly compromised in exchange for e.g. aesthetics or multiphysics requirements, but rather constitutes a design practice. Topology optimization (TO), a well-known structural design method, was focused by Sigmund [44] on the homogenization approach and the implementation of density based TO put the foundation for the nowadays methods.

The present study is done in two stages. Using a geometric model, a FE analysis is performed by using the software ANSYS 2019 R2 [21, 45, 46] in order to obtain the first natural frequency of a mounting bracket used for an industrial robotic arm. Starting from this initial design, in the first stage is obtained an optimized lattice model which becomes the basis of a second study, in which a homogenized model with variable material properties specially defined with this type of structural component is created [47]. The path from lattice optimization to model homogenization is a solution as to simplify the numerical calculations and reduce costs.

A comparison between the results on the increase of the fundamental frequency and decrease of mass obtained on the two design approaches is performed [48].

5.2. Topological optimization of a support for a robotic arm (case study)

In general, topological optimization is based on a set of loads and boundary conditions provided by one or more previous analyzes. The topological optimization analysis itself may be preceded by a structural analysis or several coupled structural analyzes. Loads and boundary conditions defined in the upstream analysis are used to create an optimized structural component based on the objectives and constraints specified in the topological optimization analysis [49]. It is necessary to use the same loads and boundary conditions in the validation analysis of the optimized design.

Therefore, in this study, a specific mounting bracket for an industrial robotic arm is analyzed. Thus, the 3D geometric model of the support is obtained first. Classical geometric modeling methods are used to obtain the 3D geometric model (Fig. 5.1). The mounting bracket analyzed and optimized in this study is fixed in the six holes at its base (Fig. 5.2).

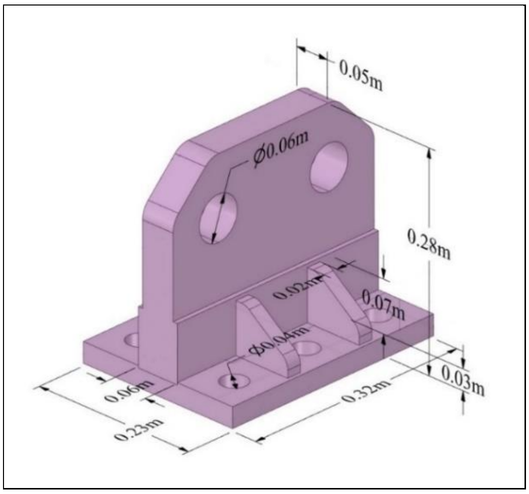


Fig. 5.1. Initial geometric model

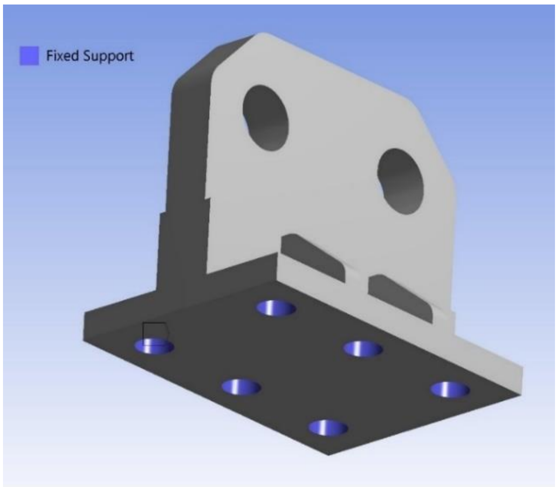


Fig. 5.2. Boundary conditions

The modal analysis is performed in order to assess the occurrence of the resonance phenomenon. In this case we are particularly interested to obtain the value of the first natural frequency (fundamental one), so that in the process of lattice optimization to obtain an improved design model for increasing this value and, in the same time, to minimize the mass of the bracket. Secondly, a homogenization analysis is meant to obtain an alternative value of the first natural frequency and compare it to the one resulting from the lattice type model.

For the initial model is assigned a metallic homogeneous and isotropic material at ambient temperature having a linear elastic behavior, with the following properties: density: $\rho = 7850$ kg/m3, Young’s modulus E = 210000 MPa, Poisson's ratio $\nu = 0.3$, tensile and compressive yield stress equal to 250 MPa, and ultimate tensile strength of 460 MPa.

The first main design approach starts with obtaining the geometric model. This model is then discretized and used in the modal analysis, in order to obtain the natural frequencies and the corresponding shape modes. Also, in this stage is performed the lattice optimization which, in this case, aims to maximize the value of the first natural frequency.

Based on the results from the topological optimization analysis, the corresponding lattice structure is generated, which allows this objective to be achieved, together with the mass minimization.

5.3. Initial model

When performing a lattice optimization, it is generally recommended to minimize the mass as an optimization objective, with restrictions specific to the analysis preceding it. For this reason, it is often necessary to set more stringent design requirements than in the case of standard topological optimization. Since it is not always possible to accurately estimate the yielding of the optimized component, it may be necessary to start the optimization with increasingly tighter constraints before obtaining a desired result from the lattice optimization.

The geometry is not complicated, so a good quality finite element model is easily obtained, by considering: finite element type Solid 185 of the first order which allows several geometric shapes and thus a hybrid finite element model of equal element size of 8 mm was generated [21, 45].

5.4. Modal analysis

Determination of the natural frequencies and the corresponding modes shape of the structural components can be realized through the modal analysis. Modal analysis is a linear analysis. Any nonlinearities, such as material plasticity or nonlinear contacts, are ignored, even if defined [45].

First 6 natural frequencies were extracted for the unoptimized model, having their values as presented in Table 5.1. Results obtained for a mesh size of 4x4x4 mm are also presented in this table. As mesh size is reduced the natural frequencies decrease a little bit, with no more than 2%, but we should mention that a warning was given during solving as mesh size was too small.

It is observed that the first natural frequency has a value of 839 Hz (Fig. 5.3) with a maximum total deformation at the top of the bracket of 14.2 mm.

Table 5.1. First 6 natural frequencies
for an unoptimized bracket model

Frequency [Hz]	Unoptimized model	
	8x8x8 mm	4x4x4 mm
1	839.1	827.4
2	1471.9	1454.8
3	1532.9	1504.0
4	2808.1	2773.7
5	3110.9	3045.7
6	3710.9	3671.5

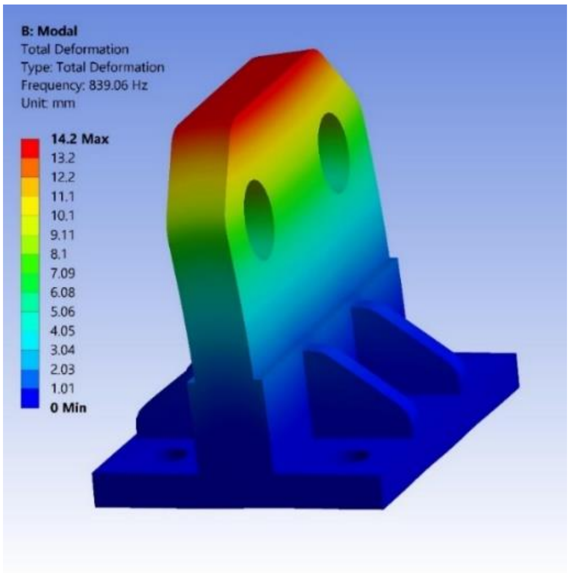


Fig. 5.3. Mode shape corresponding to
the first natural frequency

5.5. Lattice optimization

A special type of the topological optimization process is the lattice type optimization, considered here as being in the first stage of analysis. In this type of optimization, the model is "filled" with a structure optimized with lattices, the solid elements being replaced by the interconnected lattices forming a common body. When lattice optimization is performed, one can, for example, maximize rigidity or minimize the mass as an optimization objective, with the same optimization constraints available as in the case of standard topological optimization.

In this study, the main objective is to maximize the value corresponding to the first natural frequency, followed by the requirement to retain a minimum of 40% of the original support mass. The material assigned to the model and the boundary conditions are the same as for the modal analysis of the unoptimized model. The optimization region is highlighted in blue in Fig. 5.4 and will be transformed into a lattice structure; the mounting holes are not part of the optimization region and are highlighted in red in the same figure.

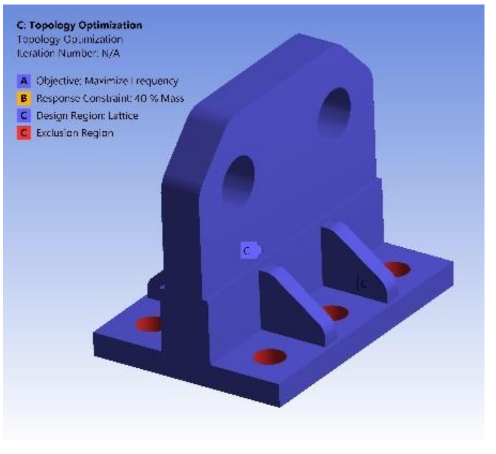


Fig. 5.4. Optimization region of the bracket and the mounting holes

It is used a cubic lattice cell, the size of lattice cell being identical to that used for the finite element (8x8x8 mm), although this is not mandatory. For the solution, a maximum threshold of 500 iterations is initially set, but as it is not a very large model the solution is obtained after only 30 iterations. The new structure of the bracket is shown in Fig. 5.5 for a regular cube lattice (cubic cell).

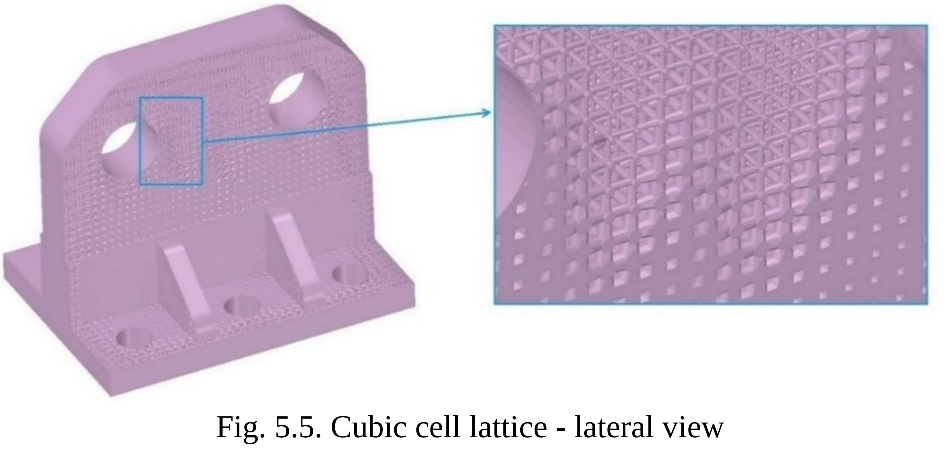


Fig. 5.5. Cubic cell lattice - lateral view

After solving the lattice optimization analysis, the new material distribution can be observed in Fig. 5.6. In the red area, practically no material is removed and as it goes to the blue area more and more material is removed. In fact, after performing the lattice optimization results are automatically converted in Ansys SpaceClaim Direct Modeler [46] in an “stl” format file and the lattice geometry is generated, as to be eventually used in 3D printing. It should be also mentioned that the initial geometry of the bracket was modeled also in Ansys SpaceClaim Direct Modeler.

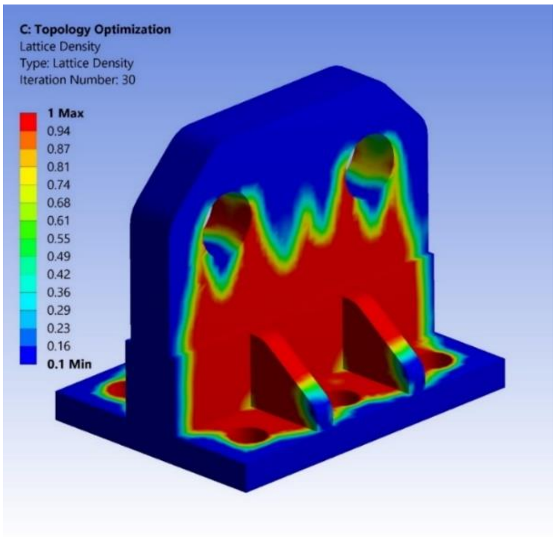


Fig. 5.6. Material distribution after solving the lattice optimization analysis

By notating the local orthogonal system of principal axes for each cell with 1, 2, and 3 the corresponding longitudinal Young’s moduli *E* and the transversal moduli *G* are practically the same on all three directions for the cubic cell (Table 5.2) and the midpoint cell (Table 5.3). Some minor differences resulted for the relative Poisson’s ratios, in most cases at the fourth decimal.

For a cubic cell and volume fraction 0.9 only a small quantity of material is removed from the center of the cube. For the midpoint cell and volume fractions of 0.8 and 0.9 it is almost impossible to detect the removed material (Table 5.3, top figures), so the midpoint cell is a little bit stiffer than the cubic cell, as *E* values are also a little bit greater for all volume fractions.

Table 5.2. Sets of elastic constants for cubic cell size of 8x8x8 mm and different lattice volume fractions

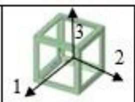
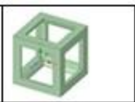
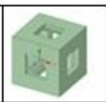
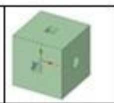
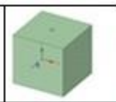
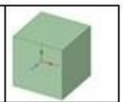
Lattice volume fraction										
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
E_1 [GPa]	8.20	18.2	30.1	44.0	60.3	79.7	103.0	130.020	162.0	200.0
E_2 [GPa]	8.20	18.2	30.1	44.0	60.3	79.7	103.0	130.020	162.0	200.0
E_3 [GPa]	8.20	18.2	30.1	44.0	60.3	79.7	103.0	130.020	162.0	200.0
G_{12} [GPa]	0.199	1.06	3.04	6.53	12.0	19.9	30.7	44.589	61.2	76.9
G_{23} [GPa]	0.199	1.06	3.04	6.53	12.0	19.9	30.7	44.589	61.2	76.9
G_{31} [GPa]	0.199	1.06	3.04	6.53	12.0	19.9	30.7	44.589	61.2	76.9
ν_{12} [-]	0.056394	0.085389	0.11149	0.13594	0.1619	0.1896	0.22001	0.25079	0.28013	0.3
ν_{13} [-]	0.056452	0.085419	0.11107	0.13612	0.16145	0.18954	0.21996	0.2514	0.28031	0.3
ν_{23} [-]	0.056452	0.085419	0.11108	0.13612	0.16147	0.18955	0.21996	0.25137	0.2803	0.3
ρ [kg/m^3]	785	1570	2355	3140	3925	4710	5495	6280	7065	7850

Table 5.3. Sets of elastic constants for midpoint cell size of 8x8x8 mm
and different lattice volume fractions

Lattice volume fraction										
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
E_1 [GPa]	8.27	18.5	30.7	45.2	62.5	83.1	107.0	134.0	164.0	200.
E_2 [GPa]	8.27	18.5	30.7	45.2	62.5	83.1	107.0	134.0	164.0	200.
E_3 [GPa]	8.27	18.5	30.7	45.2	62.5	83.1	107.0	134.0	164.0	200.
G_{12} [GPa]	0.216	1.19	3.36	7.20	13.0	21.2	31.9	44.3	59.3	76.9
G_{23} [GPa]	0.216	1.19	3.36	7.20	13.0	21.2	31.9	44.3	59.3	76.9
G_{31} [GPa]	0.216	1.19	3.36	7.20	13.0	21.2	31.9	44.3	59.3	76.9
ν_{12} [-]	0.057745	0.087889	0.11432	0.14051	0.16796	0.19726	0.2261	0.25105	0.27445	0.3
ν_{13} [-]	0.057741	0.087860	0.11426	0.14053	0.16797	0.19727	0.22582	0.25108	0.27441	0.3
ν_{23} [-]	0.057741	0.087860	0.11426	0.14053	0.16797	0.19727	0.22584	0.25108	0.27441	0.3
ρ [kg/m^3]	785	1570	2355	3140	3925	4710	5495	6280	7064.9	7850

The lattice model is an input data for the modal analysis which will provide new natural frequencies. When using a cubic cell, the first natural frequency has now a value of 1227 Hz, which increased considerably compared to the initial value of 839 Hz, this being in fact the main objective of optimization analysis. The total deformation increased to 59.8 mm compared to 14.2 mm for the solid bracket.

5.6. Homogenization of the lattice structure using the finite element method

3D printing allows the generation of parts with complex microstructures, sometimes different scales being present for a component. The ratio between the involved length scales can be significant, and by using a single finite element model the problem of length scale differences (when reducing the cell size) will produce significant computation challenges. In such situations the standard approach is homogenization [47]. Based on the assumption of a separation of scales, numerical homogenization is often used to model the lattice structures by means of equivalent material properties, as done by Jansen and Pierard [50]. In their work an industrial bracket is also used as a case study to analyse the efficiency of homogenization.

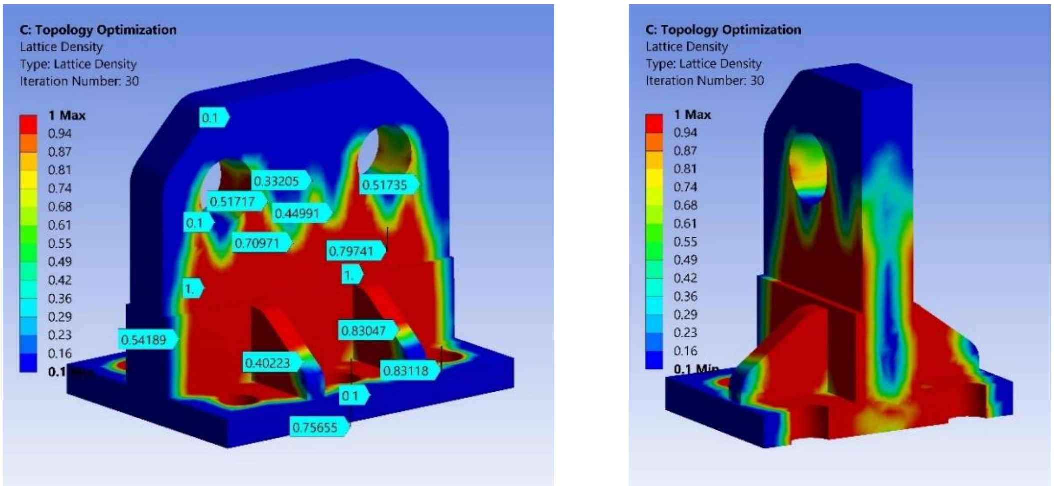
In the present study, a single computationally complex pre-processing step is performed, which leads to a homogenized material with variable characteristics and results in a macroscopic simulation that is significantly less computationally expensive. In all the simulation methods there is a hypothesis of separation of the scale.

The microscale structures must be significantly smaller than the macroscale ones. If this assumption is overrun, the micro-scale and the macro-scale cannot be modeled independently. However, this assumption is reasonable for both composite materials and lattice structures and it is used in many calculations [51].

In this second design approach of the present study a homogenized model is obtained based on which a new modal analysis is performed. In this analysis, boundary conditions and finite element model characteristics (method, element type and size, etc.) are used identical to those used in the initial modal analysis.

Therefore, the second main stage of this study begins with the design of a material model whose properties for a cubic cell vary according to the lattice density. To obtain such a material, it's generated a file containing the nodal coordinates and the lattice variable density values corresponding to them, local values being shown in Fig. 5.7. The relative density is indicated in few points in the picture as to illustrate its distribution over the surface and section of the mounting bracket.

The input data are used in a new modal analysis that will provide the natural frequencies, the first of them being the fundamental one, its increase being the main objective of the optimization process.



(a) (b)

Fig. 5.7. Local values of relative lattice densities: lateral view (a), section of the bracket (b)

After performing the modal analysis in which the homogenized model is generated starting from a cubic cell, it is established that the first natural frequency has a value of 1366 Hz (Fig. 5.8), which increases considerably compared to the initial value of 839 Hz for the unoptimized model, this being in fact the main objective of optimization analysis. Total maximum displacement became now 49.4 mm, being a little bit reduced from the value obtained for the lattice model with cubic cell.

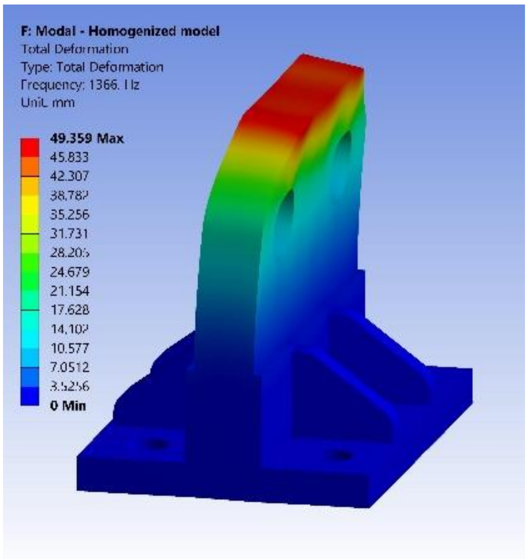


Fig. 5.8. The first natural frequency for homogenized model

Table 5.4 gives the obtained values for the first six natural frequencies obtained for the unoptimized model and the optimized model for which the two design approaches were used: lattice and homogenization. The finite element, respectively cubic cell sizes were 8x8x8 mm and 4x4x4 mm. For the smaller cell size, the lattice simulation didn't work. It is interesting to notice that for the homogenized model the first two natural frequencies increased when reducing the cell size and for the next three slightly decreased.

Table 5.4. Comparison on the natural frequencies for the unoptimized and optimized models of the mounting bracket obtained for a cubic cell

Frequency [Hz]	Unoptimized model		Optimized model			
			Lattice		Homogenization	
	8x8x8 mm	4x4x4 mm	8x8x8 mm	4x4x4 mm	8x8x8 mm	4x4x4 mm
1	839.06	827.44	1227	-	1366	1405
2	1471.9	1454.8	1893.1	-	1560.1	1579.8
3	1532.9	1504	2082.8	-	1658.5	1618.8
4	2808.1	2773.7	2342.7	-	2192.1	2147.4
5	3110.9	3045.7	2575.8	-	2389.6	2332.6
6	3710.9	3671.5	2718.7	-	2553.5	2563.1

For different cell topologies of sizes 8x8x8 mm the first six natural frequencies obtained by using homogenization are presented in Table 5.5. The fundamental frequency increases as the lattice cell is becoming more rigid.

Table 5.5. First six natural frequencies obtained for different topologies of lattice cells

Frequency [Hz]	Homogenized optimized model			
	Cubic cell	Midpoint cell	Diagonal cell	Crossed cell
1	1366	1383.3	1465.8	1483.4
2	1560.1	1586.7	1879.3	1845.7
3	1658.5	1688.5	1963.3	1969.4
4	2192.1	2238.3	2612.5	2633.8
5	2389.6	2447.1	3072.4	3048.
6	2553.5	2621.8	3115.3	3125.4

Therefore, we performed the modal analysis for the lattice structure in two ways: in the first approach, the lattice structure obtained from the topological analysis was used as a model and in the second approach a homogenized model was used. In both cases the objective of the optimization process was reached, which consisted of increasing the fundamental frequency and, at the same time, the optimization restriction which consisted in minimizing the mass was respected. The weight of the bracket was reduced by more than half, from 45.5 kg to only 21.77 kg after optimization (lattice and homogenization gave practically the same mass value) with cubic cell as presented in Table 5.6. Also, from this table it can be noticed that when using a midpoint cell, the mass has a minimum of 18.22 kg. If a diagonal or a crossed cell are used the structure is more rigid and the mass increases accordingly.

Table 5.6. Mass of mounting bracket established after homogenization for different types of lattice cells

	Homogenized optimized model			
	Cubic cell	Midpoint cell	Diagonal cell	Crossed cell
Mass [kg]	21.77	18.22	29.50	27.10

To conclude, the resulting fundamental frequencies obtained for the optimized model were: 1227 Hz for the lattice model and 1366 Hz for the homogenized one with lattice cubic cell or 1383 Hz for a midpoint cell.

There is a difference in the values obtained for the fundamental frequency between the two optimization calculation methods. This difference may be due to the level of refinement of the finite element model for the lattice structure, as in this case a refined mesh was not possible to be used.

It is also very difficult to generate a mesh for a lattice structure, requiring a high-performance computer and a long solving period. For this reason, in many finite element analyses, the use a homogenized model is preferred.

5.7. Conclusions

Topology optimization helps us to design durable and lightweight components for any application. We can define objectives easily and apply controls to ensure that manufacturing requirements are met, minimum material thicknesses are set, and exclusion areas are defined. Thus, the topological optimization leads to a reduction of the production costs, being used a minimum quantity of material necessary to meet the operating standards in optimal conditions and to increase the life of the product.

In a first stage, lattice optimization was used as an extended step from topology optimization, effective for practical engineering applications involving 3D printing. In the first step of this work an initial model of a mounting bracket considered as a case study was established through a finite element analysis. The boundary conditions were defined and the Block Lanczos algorithm was used for computing the first six natural frequencies in a modal analysis. Also, the mass of the bracket was established. The lattice optimization method enabled us to compute an optimal variable density lattice distribution in the model. A new geometrical model was generated automatically by using the ANSYS software customized for lattice structures and cubic cells of two sizes were used. The 8x8x8 mm cell led to an increase of the fundamental frequency from 839 Hz for the unoptimized model to 1227 Hz for the optimized one, and mass was reduced from 45.5 kg to only 21.77 kg, less than half. The smaller cubic cell of 4x4x4 mm could not be used effectively.

In the second stage of the study, the model was reanalyzed and homogenized, and a new modal analysis was performed using the same conditions as in the first two analyses, on the unoptimized model, respectively on the lattice model, but this time a specially designed material was used for the lattice configuration as having a variable lattice density, so the modal analysis could be done quicker and easier. Four types of cells were used. Cubic cell as for lattice optimization and three other cell types: midpoint, diagonal and crossed, all of them being available in ANSYS. The homogenization procedure increased the stiffness of the bracket by using cells with different volume fraction which generate a locally orthotropic material and therefore the fundamental frequency increased to 1366 Hz by using the same cubic cell. By changing the local stiffness through the cell type, best results were obtained with a midpoint cell as frequency was increased to 1383 Hz and mass reduced to 18.22 kg.

The obtained results can give confidence for the use of a lattice type topology optimization, but care should be taken as not to reduce too much the cell size, as the numerical solution may become tedious or ineffective. Homogenization proved to be more effective for the optimization in analyzing the modal response of the mounting bracket, by increasing the fundamental frequency and reducing its mass.

Chapter 6. Additive Manufacturing

6.1. Introduction

The late ‘80s began with the development of technologies for forming 3D objects, not according to traditional methods of removing additions (turning, milling, erosion machining) or changing the shape of the preform (rolling, molding), but using a method of adding material or changing physical properties in the material processing stage.

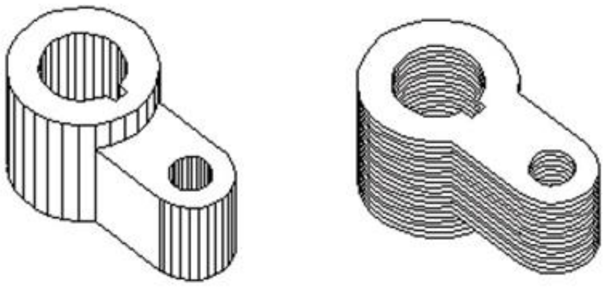
There are now advanced technologies for layer-by-layer forming of 3D objects according to electronic prototyping (CAD). These technologies are known as Rapid Prototyping (RP), generically called *additive manufacturing* (3D printing).

Rapid prototyping technology has found its use in various industries such as machine building, electronics, medicine, etc., wherever complex mechanisms are created and a series of experimental models and mock-ups of parts are manufactured, requiring a longer design and manufacturing time.

At present, various rapid prototyping systems are in use that produce models according to different technologies and from different materials based on the layer-by-layer method of forming models and consist of:

- Taking the 3D model from the CAD program (Fig. 6.1 (a))
- Dividing the 3D model into layers using a program (Fig. 6.1 (b))
- Forming the split part layer by layer from bottom to top until the actual model is obtained

The layers are formed from the bottom up on top of each other and are linked together. The formation of the prototype is carried out according to the data contained in the split model (Fig. 6.1 (b)) [52].



(a) (b)

Fig. 6.1. The principle of creating the actual model: taking the 3D model from the CAD program (a), dividing the 3D model into layers (b) [52]

Rapid prototyping equipment appeared on the market in 1987 when 3D Systems launched the stereolithography machine. Currently, there are many other companies active in this field.
We can mention some of the most popular rapid prototyping methods:

- SLA or SL – Stereolithography
- FDM – Fused Deposition Modeling
- SLS – Selective Laser Sintering
- LOM – Laminated Object Manufacturing

6.2. Advantages of 3D printing compared to traditional processes

The 3D printing process does not require molds due to the manufacturing method, the material is added in successive layers. 3D printing has several major advantages over traditional manufacturing methods:

a. Reducing the time of completion
Because of their complexity, modern molds sometimes take several weeks to be completed. Not only do 3D printed objects not require molds, but even the most complex designs can be 3D printed in one piece, so no assembly process is required, once the backing material has been removed, we have the finished piece.

b. Speed
Because this process does not involve any type of die, the manufacturing time is the time it takes to 3D print the product. Thus, a small object can be produced and delivered in just a few hours. If the desired object has a complex structure, using traditional technologies, just making the mold would take weeks, and if the process also requires assembly, production time and costs increase accordingly.

c. Cost by volume only
One of the most exciting qualities of 3D printing is that the volume of material used is the only basis for calculating the cost of manufacturing. Since 3D printing technology consists of “building” the ready-assembled object or mechanism by depositing successive layers of material, the complexity of the work does not matter and no other adjacent work and processes are required. Production costs will thus be related only to the size of the object, regardless of its complexity.

d. Making shapes impossible for traditional technology
3D printing has the incredible ability to make perfectly functional mechanisms in a single operation, without further assembly and post-production processes. Mechanisms such as ball bearings, chains, even entire gearboxes or motors can be printed in a single technological process.

e. High precision
The 3D printer’s compact and robust construction and material composition gives the object excellent mechanical rigidity and the ultra-compact extruder ensures precise and uniform material application with an accuracy of 0.15 mm/layer.

6.3. Basic technologies of rapid prototyping

The operation of rapid prototyping systems is based on a chemical photopolymerization process, where the liquid polymer solution is transformed into a solid polymer under the influence of ultraviolet rays.

Other rapid prototyping systems operate on the basis of thermal processes through which physical models are obtained. This technological process involves injection molding of thermoplastic materials, which, by layering, form the physical model. Under the influence of the thermal processes, the powder-like material solidifies and the successively deposited layers are bonded together.

6.3.1. Stereolithography

Rapid prototyping technologies are based on stereolithography. Stereolithography (SLA or SL) was pioneered by 3D Systems Company in 1987 and today more than 500 of its stereolithography systems (Stereo Lithography Apparatus – SLA) are promoted and used by companies worldwide. Stereolithography systems produce high-precision photopolymer objects based on 3D CAD models.

The stereolithography process involves local phasic change of the medium (transition from liquid to solid state) in the indicated polymerization volume. The purpose of photopolymerization is the formation of interaction capacities (radicals, ions, active particles) by laser beams in the active liquid medium, which react with monomer molecules, initiating the growth of chain polymers. As a result of this process the phasic change of the medium takes place, i.e., at the interaction between the laser beam and the active liquid the solid polymer is obtained [53].

The laser is the main working element of stereolithography which, step by step, generates the shape of the object section. The liquid photopolymer solidifies where it is in direct contact with the laser beam. The advancing platform, on which the part grows, is placed below the surface with the photopolymerization substance by a distance equal to the thickness of the first layer. An image is formed on the photopolymerization surface, which corresponds to the first section of the object. The solid polymer film is formed in the laser beam range. After the formation of the first layer, the platform with the formed film is moved down to the distance equal to the thickness of the next layer. The new layer of material is placed on the formed surface and the image corresponding to the second section of the part is projected onto the surface with the photopolymerization substance. The advancing platform is then moved to a distance equal to the thickness of the next layer and the process is repeated automatically until the part is obtained. Once the last layer is obtained, the platform moves up to the first layer where the printed part is taken off the platform, then the liquid substance of the photopolymer is removed from the surface of the part and finally the part is dried [54].

Stereolithographic printing technology was proposed by Charles Hall in 1984. The stereolithography machine was launched on the market by the company 3D Systems Inc. in the United States and the first machines appeared in 1988. The structural diagram of the implementation of the technology is shown in Fig. 6.2.

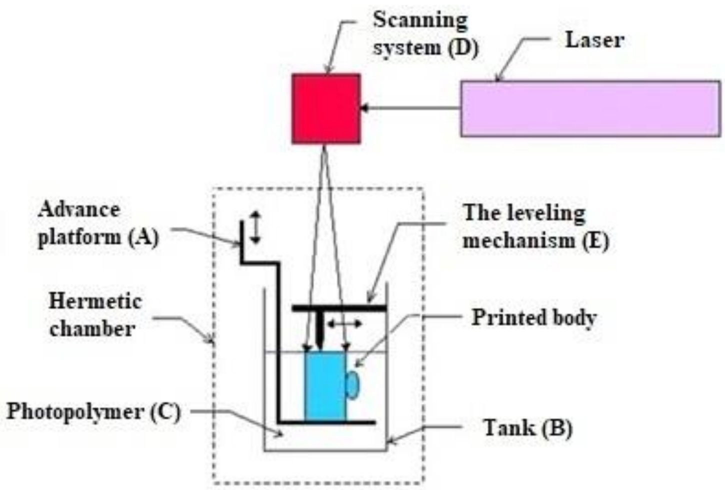


Fig. 6.2. Diagram of the stereolithography process

The process continues step by step until the piece is completed. Some part designs may have small deviations and need additional support during the printing process. Supports are created during the 3D modelling of the part using various computer-aided engineering programs.

The supports allow:

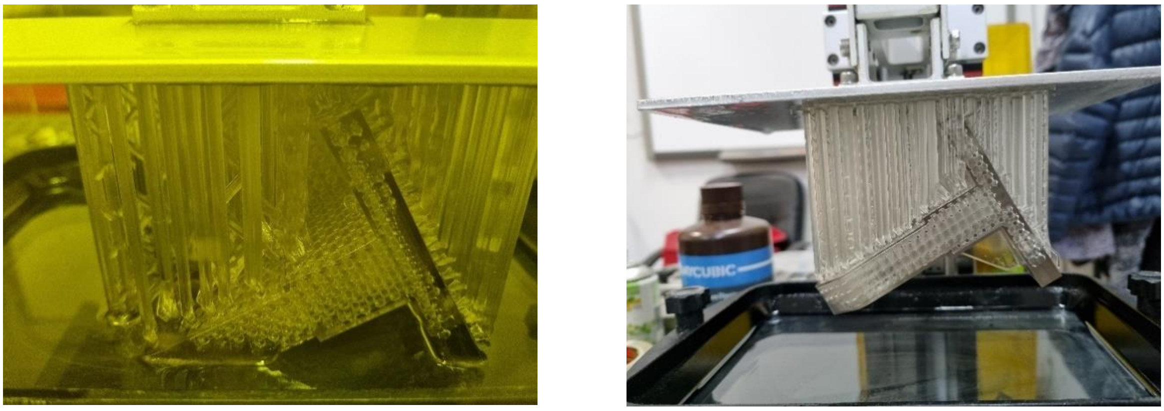
- stabilization of the part
- prevents disassembly of layers in difficult areas of the part

- corrects some layer deviations
- easy removal of the printed part from the workroom
- manufacturing of complicated multi-part designs

When manufacturing the prototype, one of the most important goals is precision. Analysis of research and experiments has identified three main factors that influence the accuracy of prototyping:

- system parameters
- layer-by-layer model generation
- the material used to make the part

Fig. 6.3 (a), (b) presents sequences of the 3D printing process.



(a) (b)

Fig. 6.3. 3D Printing of a lattice support structure:
intermediate stage (a), final stage (b)

6.4. Numerical simulation of the additive manufacturing process of a mounting bracket

Some finite element method-based programs allow topology optimization studies, which involve minimizing or maximizing objective functions while imposing specific design constraints, ultimately determining an optimal design for a structural component [55, 56].
A particular case of the topology optimization process is lattice optimization. When performing such optimization, one can for example maximize stiffness or minimize mass as an optimization objective, with the same optimization constraints available as in standard topology optimization. When solving a lattice optimization, it is generally recommended, if the software used allows, to include simulation of the additive manufacturing process. This is because lattice structures are often manufactured in this way. In the present study such a simulation is highlighted [57].

6.4.1. Objective of the study

The simulation of the additive manufacturing process uses the optimized geometric model resulting from the case study presented in Chapter 5. The simulation consists of a customized transient thermal analysis for additive manufacturing. The finite element method is used as numerical calculation method.
The objective of this study is to perform a preliminary verification of how the real manufacturing process would proceed.
The complete diagram of the numerical study is shown in Fig. 6.4.

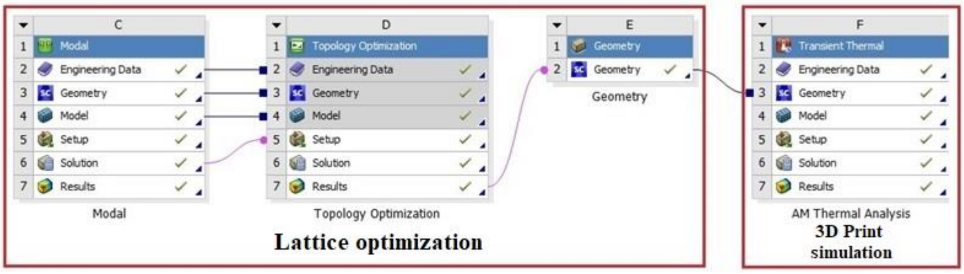


Fig. 6.4. Numerical study diagram

6.4.2. Calculation assumption

After performing the lattice topology optimization, the corresponding lattice structure automatically results, and this geometric model is then used in the simulation of the additive manufacturing process [39]. As mentioned above, this simulation is thermal, time-dependent and consists of two major steps:

- In the first stage, the consecutive deposition of the element layers takes place at temperatures specific to this type of manufacturing process and taking into account all the technical details that it involves
- The second stage consists in simulating the cooling of the printed model and its solidification.

In this simulation, input data specific to the Powder Bed Fusion (PBF) process were used and the material used in the simulation is Inconel 718.

6.4.3. Geometric model

As mentioned above, in the simulation of the additive manufacturing process, the geometric model of a mounting bracket resulting from the lattice topology optimization presented in Chapter 5 is used, which is a specific mounting bracket for an industrial robotic arm. In Fig. 6.5 - 6.6 the lattice-type geometrical model is shown in cross-section and longitudinal section respectively.

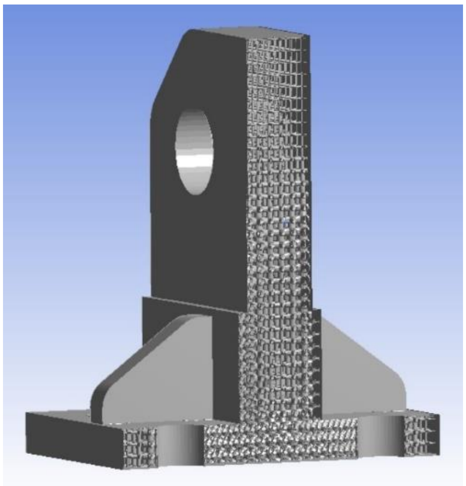


Fig. 6.5. Geometric model - cross-section

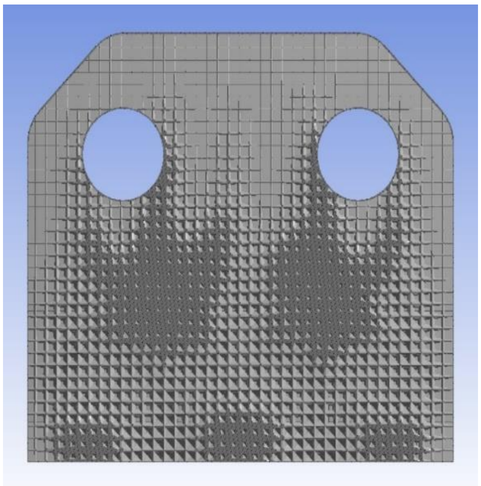


Fig. 6.6. Geometric model - longitudinal section

6.4.4. Discretized model

Since the geometric model has a complex structure, it is difficult to obtain a good quality discretized model by a classical method. For this reason, the Cartesian method has been used to discretize the printed part and the support on which it rests. An element size of 1 mm was imposed. It is important to note that the discretized model for the support was automatically generated by the program without the need to create a geometric model for it beforehand. Nor could this have been possible due to the complexity of the "lattice" structure.

The Cartesian method generates a structured, uniform mesh consisting only of cubic elements. The size of the element must be small enough to best fit the geometric model. Discretized models used in explicit dynamic simulations or models with regular geometric layout are good examples for which this discretization method could be used. It is also recommended for simulating the printing process in additive manufacturing [58]. A classical mesh was generated for the discretization of the base plate. In Fig. 6.7 the fabricated part is highlighted in red color, the base plate is dark blue and the part support is colored in turquoise blue.

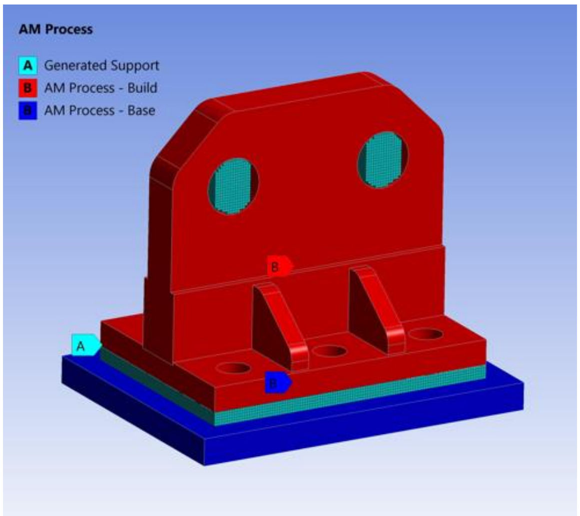
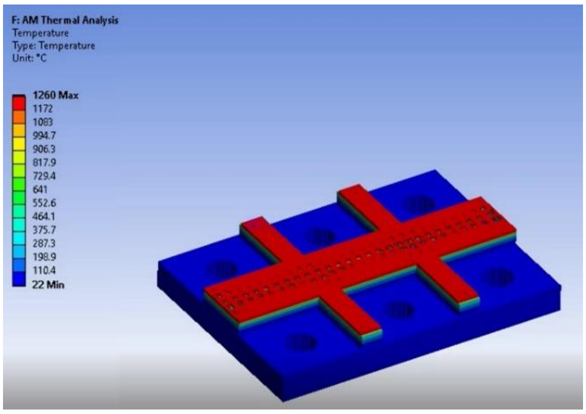


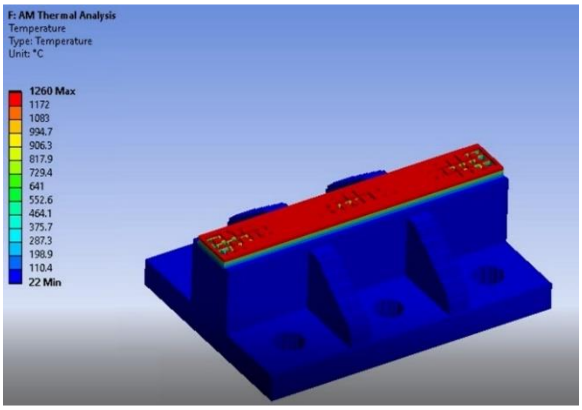
Fig. 6.7. Full model used in AM simulation

6.4.5. Results

Following the solution of the transient thermal analysis, it is possible to observe the part generation process in different intermediate stages as well as the temperature distribution. In the red areas the temperature reaches the maximum value and as one moves towards the blue area, the part cools down to the room temperature (22 ºC) because this value was imposed when the cooling conditions were defined (Fig. 6.8 (a), (b)).



(a)



(b)

Fig. 6.8. Footage from the AM simulation

6.5. Conclusions

Creating a new type of product is a lengthy and complex process that requires several stages of design and analysis before mass production begins. These stages can be completed more quickly by implementing 3D modelling methods. Modern computer-aided design systems can considerably reduce the time needed to design and model a new product and cut costs. However, the problem of manufacturing the first real model or an element of a part that has a more complex shape remains at the forefront, as the creation of the technological process of manufacturing the part and the elements that contribute to this process requires significant expenditure.

In the specific working process of a new project, especially in the initial design stage, it is difficult to detect shortcomings and mistakes, having only the 3D geometric model in the computer. Having the actual model of the part can help to detect and remove various errors and also improve the methods of the design process. The prototype part can be used as a concept model for visualization and analysis. The prototype allows designers to carry out some functional tests and possible modifications.

Prototype models help to reduce design costs and time to market by detecting possible errors at an early stage, broaden the dialogue between the designers and the product purchaser, and last but not least reduce the time to market.

Chapter 7. Verification of modal response for a mounting bracket using numerical simulations and experimental tests

7.1. Introduction

The numerical study is carried out in two steps. Using a geometric model, a finite element analysis is performed using ANSYS 2019 R2 software [21, 45, 46] to determine the first natural frequency of a mounting bracket. Starting from this initial design, an optimized lattice model is obtained in the first step, which becomes the basis for a second study, in which a homogenized model is created with variable material properties specifically defined for this type of structure [47]. The path from optimizing the lattice structure to homogenizing the model is a solution for simplifying numerical calculations and reducing costs. Finally, a comparison is made between the results obtained by numerical simulation and those obtained experimentally.

7.2. Description of the case study

In this study, a mounting bracket similar to the one presented previously in Chapter 5 is analyzed, but with reduced dimensions so that it can be printed and tested experimentally more easily (Fig. 7.1). This mount is fixed in the bottom six holes and braced to the base.

Modal analysis is performed to assess the occurrence of the resonance phenomenon. In this case we are interested in obtaining the value of the first natural frequency (fundamental frequency), so that in the process of optimizing the lattice structure we obtain an improved model that allows increasing this value and at the same time we want to minimize the mass of the support.

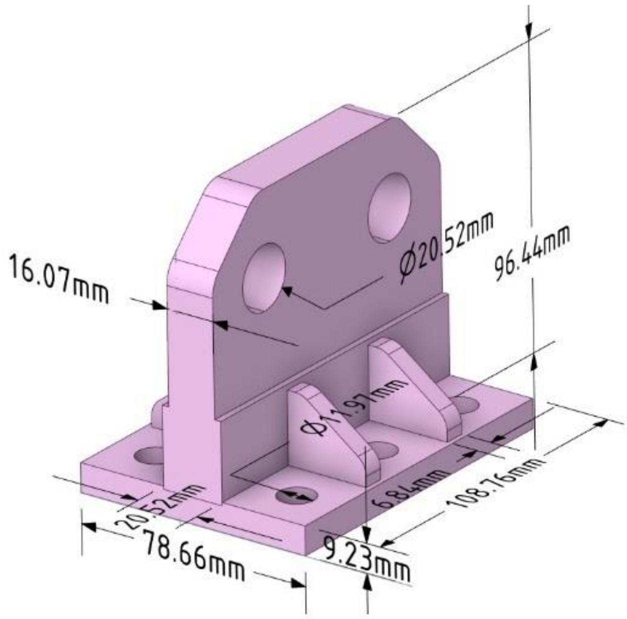


Fig. 7.1. Initial geometric model

The numerical study therefore consists of two main design stages which in turn contain several sub-stages:

- Optimization analysis of the lattice structure consisting of: creation of the initial geometric model; definition of all input data including boundary conditions and material properties (technical data); topology optimization;
- Homogenization analysis taking into account: same technical data as in the first case, configuration of external data, i.e., a “csv” file which is not directly generated by ANSYS Mechanical [45] and which contains all the information about the nodes combined in Excel with the corresponding values of the lattice density; in turn, this “csv” file is combined with the one resulting from ANSYS Material Designer [47] and thus results in the homogenized model; evaluation of the homogenized model.

A homogeneous and isotropic material (PLA resin) at room temperature with linear elastic behaviour is used for the initial model with the following properties: density ρ = 1180 kg/m3, Young’s modulus E = 1027 MPa, Poisson’s ratio = 0.36 and yield strength equal to 32 MPa.

As the geometry is not complicated, a good quality finite element model is easily obtained, taking into account: the first order Solid 185 finite element type which allows several geometrical shapes and thus a hybrid finite element model was generated with an element size of 3 mm.

7.3. Modal analysis

For the unoptimized model, the first 6 natural frequencies were extracted with the values shown in Table 7.1. For a better comparison of the results, the first 6 natural frequencies were also extracted for the same model but without defining boundary conditions (free model). These are also shown in Table 7.1. It can be seen that the first natural frequency for the fixed model has a value of 323 Hz and for the free model has a value of 510 Hz.

Table 7.1. The first 6 natural frequencies for unoptimized model obtained numerically with FEM

Frequency [Hz]	Unoptimized model	
	Fixed model	Free model
1	323	510
2	632	863
3	1011	1267
4	1412	1563
5	1891	1931
6	1987	2029

7.4. Lattice optimization

In this study, the main objective is to maximize the value corresponding to the first natural frequency, followed by the requirement to keep at least 50% of the initial mass of the substrate. The material assigned to the model and the boundary conditions are the same as for the modal analysis of the unoptimized model. The optimization region is highlighted in blue in Fig. 7.2 and will be transformed into a lattice structure.

The mounting holes, the base surface and the two holes at the top of the part are not part of the optimization region and are highlighted in red in the same figure.

This means that no material will be removed from these areas. The exclusion of these areas from the optimization region facilitates the additive manufacturing process of the lattice structure and also gives the part greater strength in service.

We used a cubic lattice cell, the size of the lattice cell being identical to that used for the finished element (3x3x3 mm), although this is not mandatory. For the solution, a maximum threshold of 500 iterations is initially set, but since it is not a very large model, the solution is obtained after only 18 iterations. The new support structure is shown in Fig. 7.3.

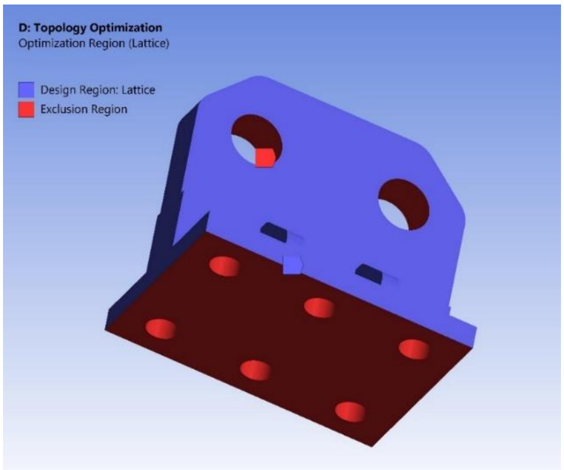


Fig. 7.2. Support optimization region and excluded areas

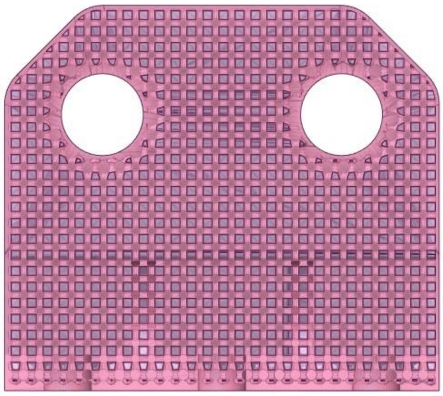


Fig. 7.3. Lattice model of the support: cross side view for the lattice network with cubic cell

After solving the lattice optimization analysis, the new material distribution can be seen in Fig. 7.4. In the red area, virtually no material is removed at all and as it moves towards the blue area more and more material is removed. In fact, after performing the lattice optimization, the results are automatically converted in Ansys SpaceClaim Direct Modeler [46] to an “stl” file format and thus the lattice network geometry is generated and finally sent to the 3D printer.

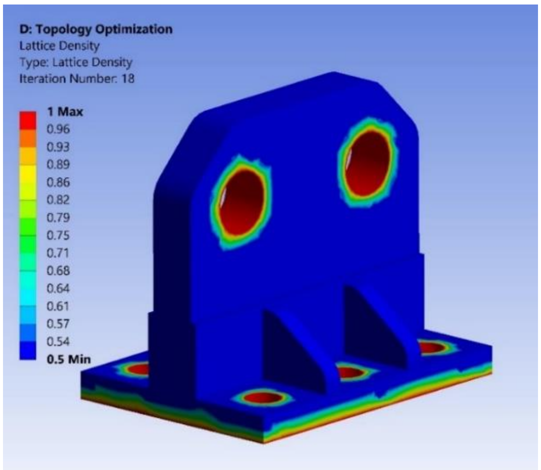


Fig. 7.4. Distribution of material resulting after solution of the lattice optimization analysis

Scoring the local orthogonal system of the main axes for each cell with 1, 2 and 3, the Young longitudinal moduli and G transverse moduli are practically the same in all three directions for the cubic cell (Table 7.2). Some minor differences resulted for the Poisson's ratio values, in most cases to the fourth decimal place.

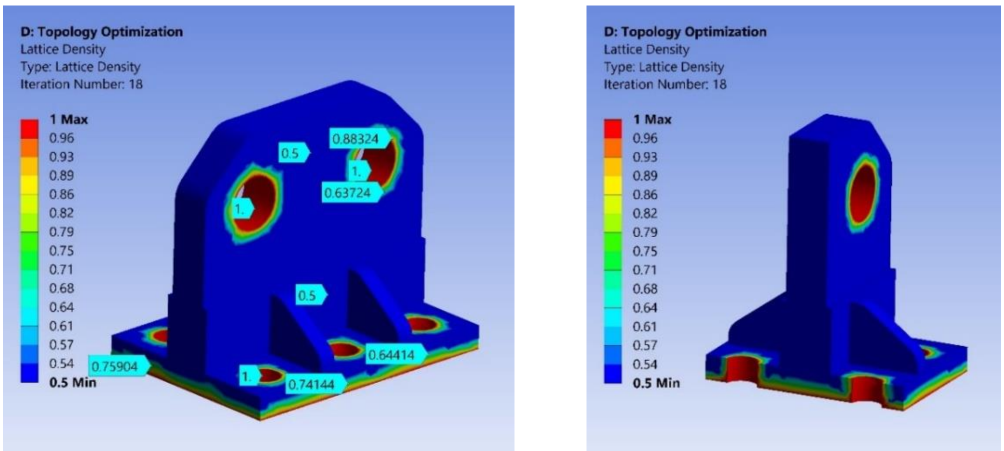
For a cubic cell and a volume fraction with a value of 0.9, only a small amount of material is removed from the center of the cube.

Table 7.2. Sets of elastic constants for the cubic cell and various lattice volume fractions

Lattice volume fraction	0.5	0.6	0.7	0.8	0.9	1
E_1 [GPa]	0.607	0.804	1.035	1.308	1.628	2
E_2 [GPa]	0.607	0.804	1.035	1.308	1.628	2
E_3 [GPa]	0.607	0.804	1.035	1.308	1.628	2
G_{12} [GPa]	0.120	0.198	0.302	0.434	0.589	0.735
G_{23} [GPa]	0.120	0.198	0.302	0.434	0.589	0.735
G_{31} [GPa]	0.120	0.198	0.302	0.434	0.589	0.735
ν_{12} [-]	0.18886	0.22191	0.25847	0.29556	0.33249	0.36
ν_{13} [-]	0.1888	0.22193	0.25754	0.29597	0.3326	0.36
ν_{23} [-]	0.18881	0.22193	0.2576	0.29593	0.33259	0.36
ρ [kg/m^3]	590	708	826	944	1062	1180

7.5. Homogenization of the lattice structure using the finite element method

The second main stage of this study begins with the design of a material model whose properties for a cubic cell vary with lattice density. To obtain such a material, a file is generated containing the nodal coordinates and their corresponding lattice density values, the local values being shown in Fig. 7.5. The relative density is indicated at several points in the image to illustrate its distribution on the surface and in the cross-section of the mounting support. The material model thus obtained is used as input data in a new modal analysis that will provide the natural frequencies, the first of which is the fundamental one, its increase being the main objective of the optimization process [48].



(a) (b)

Fig. 7.5. Local values of the lattice density: side view (a), sectional view (b)

After solving the modal analysis in which the optimized model is used, it is determined that the first natural frequency has a value of 470 Hz, which increases considerably compared to the initial value of 323 Hz for the unoptimized model, which is in fact the main objective of the optimization analysis. It is important to note that these values correspond to the fixed model.

Table 7.3 shows the values obtained for the first six natural frequencies resulting from the analysis solution for both the unoptimized and the optimized model.

Tabel 7.3. Numerically determined natural frequencies

Frequency [Hz]	Unoptimized model		Optimized model	
	Fixed model	Free model	Fixed model	Free model
1	323	510	470	693
2	632	863	918	982
3	1011	1267	1415	1373
4	1412	1563	1491	1613
5	1891	1931	2032	1972
6	1987	2029	2115	2085

Therefore, we performed the modal analysis for both the unoptimized and optimized model in two ways: in the first approach we considered the fixed (fitted) model and in the second approach the free model. In both cases the objective of the optimization process, which was to increase the fundamental frequency, was achieved and at the same time the optimization constraint, which was to minimize the mass, was respected. The mass of the substrate was considerably reduced from 0.273 kg to only 0.175 kg after optimization as shown in Table 7.4.

Table 7.4. Mounting bracket mass before and after optimization

	Unoptimized model	Optimized model
Mass [kg]	0,273	0,175

In conclusion, the resulting fundamental frequencies obtained for the unoptimized model were 323 Hz for the fixed case and 510 Hz for the free case, and for the optimized model were 470 Hz for the fixed case and 693 Hz for the free case. The geometry of the structure to be homogenized will play a role in determining the variable volume fraction of the lattice, i.e., the variable density. These sensitive attributes will closely modify the mass of the part and its fundamental frequency.

7.6. Experimental tests

The method most often used in experimental modal analysis for the excitation of a structure is the impact hammer method, also called modal hammer (Fig. 7.6 (a)). The waveform produced by the impact, given the very short time duration, is in the transient excitation regime.

The duration of the impact and the shape of the spectrum of the exciting force are determined by the mass and stiffness of the two bodies: the modal hammer and the support structure. The hammer head acts as a mechanical filter, determining the frequency range in which the vibration energy is concentrated.

The measured force is the force obtained from the product of the acceleration and the mass behind the force sensor. The actual force which excites the structure is equal to the total impact mass (including the mass of the force transducer) multiplied by the acceleration during impact. The data acquisition and processing system used in the experiment is a Pulse 7700 model made by Brüel & Kjær Company (Fig. 7.6 (b)).

The results determined in the numerical study are thus verified by experimental tests. Both the unoptimized and the optimized models were tested, fixed (fitted) in the bottom six holes and braced on the base surface. The studied support was clamped by a 15.950 kg mass in which six M8 screw holes were given. It was clamped with nuts that distributed over a larger area the stress that occurs when tightening screws. A piezoelectric accelerometer with top connector type 4384 was also mounted on the bracket. This type of accelerometer is designed for vibration testing and analysis and for general purpose high frequency measurements.

Fig. 7.6 (a) shows a sequence from the experimental test performed for the optimized model, and Fig. 7.6 (c) highlights the unoptimized model.

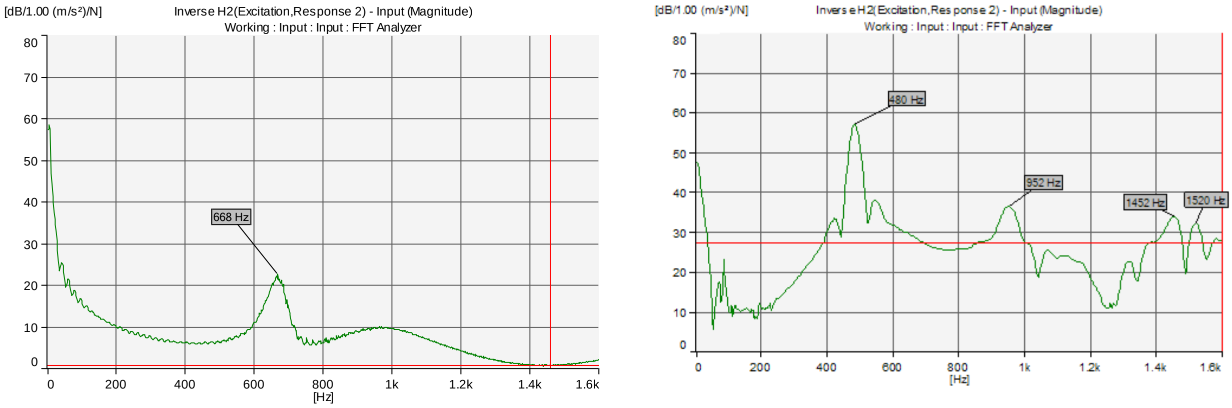
(a)

(b)

(c)

Fig. 7.6. Experimental determination of natural frequencies for the studied support: optimized model – detail (a), optimized model – overview (b), unoptimized model – detail (c)

The mechanical mobility spectrum for the fixed case (acceleration – output quantity/force – input quantity) recorded by the piezoelectric accelerometer mounted on the part is shown in Fig. 7.7.



(a) (b)

Fig. 7.7. Mechanical mobility spectrum for the fixed case: unoptimized model (a), optimized model (b)

Fig. 7.8 shows schematically how the model is clamped to determine the natural frequencies for the free case. The model is clamped to the supporting frame with flexible cables from the four corner holes. As in the clamped case, a piezoelectric accelerometer with top connector type 4384 is mounted on the part in the same location.

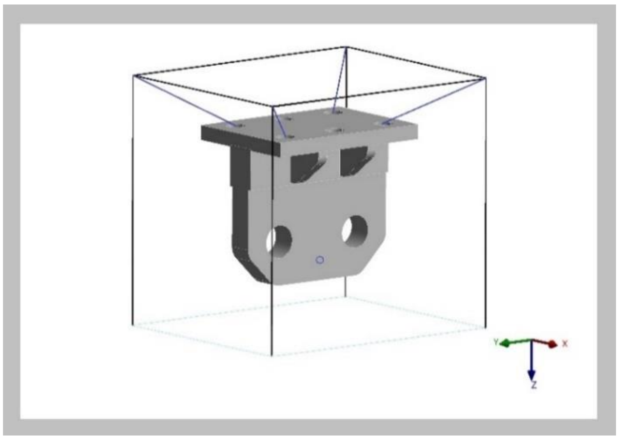


Fig. 7.8. Schematic representation of the clamping for free model testing

Table 7.5 shows the experimentally determined frequencies for both the unoptimized and optimized models corresponding to both cases (fixed and free).

Table 7.5. Experimentally determined natural frequencies

Frequency [Hz]	Unoptimized model		Optimized model	
	Fixed model	Free model	Fixed model	Free model
1	343	494	480	708
2	648	874	952	994
3	1034	1278	1452	1352
4	1435	1546	1520	1593
5	1910	1942	2087	1985
6	2015	2043	2140	2103

As you can see from Table 7.3 and Table 7.5, there are small differences between the results of the numerical study and those obtained experimentally, the latter being slightly higher in value, but these differences can be considered acceptable. They may be due both to small numerical errors and to the sometimes imprecise collection of experimental data.

7.7. Conclusions

In the first stage of the numerical study presented in this chapter, an initial model of a small-sized mounting bracket made of resin in a 3D printer was produced. The lattice optimization method allowed us to calculate an optimal lattice distribution with variable density, and the cubic cell was used. Thus, a new geometric model was automatically generated using the ANSYS software.

In the second stage of the study, based on the lattice structure obtained in the first stage, the homogenized model was produced. Thus, a specially designed material was used to equalize the lattice configuration with variable density, so that the modal analysis could be done faster and reduced computational resources were used. This equivalent model was then used in new modal analyses to determine the six natural frequencies specific to the optimized model. Two case studies were considered: one in which the model was fixed (fitted) and one in which the model was free.

Following the lattice topology optimization, the mass was reduced from 0.273 kg to only 0.175 kg and the numerically determined fundamental frequencies increased from 323 Hz to 470 Hz for the fixed case and from 510 Hz to 693 Hz for the free case. In experimental tests, the fundamental frequencies also increased from 343 Hz to 480 Hz for the fixed case and from 494 Hz to 708 Hz for the free case.

As mentioned above, the small differences in values between the results of the numerical study and those obtained experimentally may be due both to small numerical errors and to the sometimes imprecise collection of experimental data.

The results obtained can give confidence in using a lattice topology optimization, but care should be taken not to reduce the volume fraction of the lattice cell too much, as the numerical solution may become inefficient and it is also more difficult to print the part on the 3D printer. Homogenization has been shown to be effective in optimizing the modal response of the mounting bracket by increasing its fundamental frequency and reducing its mass.

Chapter 8. Final conclusions

8.1. General conclusions

The Design of Experiments (DOE) provides systematic methods to determine the relationship between factors influencing an optimization process and the outcome of that process. In other words, these methods are used to find the cause-effect relationships. This information is needed to manage the inputs to a process in order to optimize the outputs. Understanding these methods first requires knowledge of statistical tools and experimental concepts. Although “software” programs can often be used, it is important for practitioners to understand the basic concepts for proper application.

The main concerns in designing of experiments include establishing validity, reliability and replicability. For example, these concerns can be partially addressed by carefully choosing the independent variable, reducing the risk of measurement error and ensuring that the method documentation is sufficiently detailed. Related concerns include achieving appropriate levels of statistical precision and sensitivity. Based on the results obtained, the following general conclusions can be drawn:

(1) Factorial methods are considered classical DOE methods. However, they are still widely used today. These methods have the advantage that they can be used in both physical and computer experiments and can thus be combined with various numerical methods.

(2) In this paper some of the most important aspects of the use of the finite element method in mechanical engineering have been presented, with emphasis on its applicability in the optimization process of structural components. In the development stage of the calcultion model for optimization, practical aspects that are often difficult to formulate in numerical terms should also be considered. Finding a solution is done by the design engineer or other members of the technical team on the basis of intuition and experience gained over time.

(3) Optimization techniques have increasingly established themselves as very useful design tools to achieve lightweight, robust and low-cost structures. Many of these optimization techniques are based on the finite element method and the use of increasingly powerful computers. This is because, in general, optimization studies involve many experiments and also the level of refinement of the finite element model often requires the use of a powerful computer. For efficient modelling it is recommended to use some types of finite elements which first need to be analyzed both theoretically and practically, so as to know their performance and whether or not they match the design under optimization. The use of shape functions, numerical integration and the calculation of displacements and stresses are also important. The user of the finite element method must be well acquainted with all these concepts because otherwise the use of this method may lead to erroneous/inaccurate results.

(4) The response surface method and topology optimization can only be used in computer experiments. This of course also implies the use of a powerful computing machine, especially for topology optimization. Both methods use specific mathematical algorithms and also depend on the level of refinement used in obtaining the discretized model.

(5) Topology optimization is the newest of these optimization methods and has the advantage that it is easy to perform by any engineer who has a basic knowledge of the finite element method. Topology optimization is also widely used in computer-aided additive manufacturing using 3D printers, which is very common in many modern factories.

Classical design methods are not very compatible with new manufacturing processes such as additive manufacturing, which remove design constraints and open up new possibilities. The optimal shape of a part is often organic and counter-intuitive, so designing it requires a different approach. Topology optimization allows us to specify where boundary conditions and loads lie on a structural component, after which the software finds the best shape for it. Thus, topology optimization leads to a reduction in production costs, with the minimum amount of material required to meet the standards for optimal operation and increase the life of the product.

(6) With regard to the case study presented in Chapter 5, in the first step the specific boundary conditions of the studied part were defined, the number of eigenfrequencies to be extracted and the algorithm used in the modal analysis were determined, and then the custom topology optimization analysis for lattice structures was performed, resulting in a new internal configuration for the optimized model. The next step was a validation of this new model, performing a new modal analysis under the same conditions as the first one. It is important to point out that the lattice optimization resulted in a stiffer part, which was also intended, since the value of the first natural frequency increased considerably and at the same time its mass was reduced to less than half of what it had originally.

(7) 3D printers allow designers to produce a prototype in a very short time. Consequently, the prototype can be tested and reshaped quickly. Producing these parts using traditional methods takes several weeks, but using these new printing technologies reduces this time to 48 hours. The time saved thus allows the possibility of testing several variants of the components in order to develop the proposed solution as quickly as possible. 3D printing is also distinct from traditional machining techniques, which are mainly based on the removal of material by methods such as cutting, milling, drilling, etc. With 3D printing, fully functional mechanisms can be produced in a single technological process without the need for further post-production assembly processes. A 3D printer is after all another type of industrial robot, which is capable of carrying out this process under computer control.

(8) “Software” programs complement this modern, largely computer-based technological system, enabling a complete workflow from the creation of the 3D CAD geometric model to its printing. This workflow obviously also includes steps such as design optimization or design testing using numerical calculation methods. The ability to numerically simulate the additive manufacturing process is a great advantage for this workflow both in terms of time reduction and especially in terms of reducing production costs. This type of simulation generally requires a fine discretized model to respect all geometrical details, including those areas of the model that are thin or very thin. This generally leads to a discretized model with a large number of nodes and elements requiring significant computational resources. HPC computing machines are preferred in such situations.

The main objective of this paper was to carry out a detailed experimental and numerical study on the topology optimization of structural components. However, there are a number of directions that can be pursued to bring improvements to the study, namely:

(1) Improving the additive manufacturing process to print more complex lattice structures.

(2) Improving the experimental test procedure for determining the specific eigenfrequencies of the free model by introducing a more flexible clamping system.

(3) Obtain all material-specific input data used for numerical simulation of the additive manufacturing process.

(4) The possibility to numerically simulate 3D printing by as many additive manufacturing methods as possible, using discretized models that closely approximate the actual physical model.

8.2. Original contributions

The personal contributions brought to the experimental and numerical research on topology optimization of structural components are notable for:

(1) Development of a clear and efficient methodology of experimental and numerical analysis for the design and topology optimization of structural component.

(2) A comparative study of the influence of geometrical, additive manufacturing and testing parameters on the modal response of lattice structures.

(3) Development of experimentally validated numerical models for the analysis of lattice structures under modal vibrations.

(4) Conduct a comparative study of several types of lattice cells in order to determine the optimal cell type for subsequent use in both numerical and experimental studies.

(5) Qualitative evidence of the behavior of lattice structures to modal vibrations, while having the advantage of smaller masses compared to “solid” parts.

(6) Successful use of the Cartesian discretization method in conjunction with the classical discretization method for numerical simulation of the additive manufacturing process.

(7) Highlighting how to avoid the difficulties of obtaining the discretized model for a lattice structure using the equivalent homogenized model.

(8) Use of factorial optimization methods combined with the finite element method in advanced numerical simulations for determining the behavior of steel structures under static and dynamic stresses.

8.3. Future research directions

The results obtained in this PhD thesis provide important insights into the response of lattice structures to modal vibrations and at the same time constitute a starting point for several future research directions:

(1) Experimental and numerical study of the resistance of lattice structures to ordered and random vibrations, studied both separately and in combination, to verify the worst-case behavior of the structure.

(2) Experimental and numerical study of the transient dynamic response of lattice structures.

(3) Experimental and numerical study of the dynamic response of lattice structures to impact.

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