

Title: Robust and Optimal Control of
Networked and Singular Systems
PhD Candidate: Andrei Sperilă
Advisor: Cristian Oară
Co-advisor: Bogdan D. Ciubotaru

This thesis proposes theoretically sound and numerically reliable procedures for the distributed control of interconnected dynamical systems with possibly singular input-output behavior. A versatile and sparsity-oriented systemic representation is investigated and the control architectures based upon it are formalized into the Network Realization Function design framework. These representations are further developed into the System Response-Type Realizations, which enable stable and stabilizing implementations for discrete-time distributed control laws. Robustness-oriented techniques are employed to enable nominally infeasible sparsity patterns for the obtained distributed controllers, with both iterative and direct numerical procedures being developed towards this end. Finally, algebraic Riccati equation-based techniques are proposed, enabling norm optimization in the context of obtaining stabilizing controllers for singular systems.

Keywords: distributed control, dynamical networks, singular systems

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