



UNIVERSITY POLITEHNICA OF BUCHAREST

Doctoral School of Electrical Engineering

PHD THESIS

- SUMMARY -

**DISTORTED NON-SINUSOIDAL BEHAVIORS IN
NON-LINEAR THREE-PHASE CIRCUITS**

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KEY WORDS: Nonlinear three-phase circuits, Harmonic analysis, Phase separation, Hăntilă method, Controlled sources, Picard-Banach sequences in Hilbert spaces, Țugulea Theory, Power Flow, Balance of Powers, Different reactances on sequences, Nonlinear elements with controlled switching, Thyristor, Convergence Acceleration, Gibbs Phenomenon.

CHAPTER 1. INTRODUCTION

Important research on power flow in non-linear three-phase circuits was initiated by acad. A. Țugulea [1-5] and then developed by a team from the Electrotechnical Department of the Faculty of Electrical Engineering from the Politehnica University of Bucharest [6-13].

Current trends in electricity consumption and production are to reduce costs and increase energy efficiency using on large-scale power electronics [14-26]. However, non-linear circuit elements are an important source of harmonics in three-phase circuits.

1.1 DISTORTED EFFECTS CAUSED BY NONLINEAR ELEMENTS IN THREE-PHASE ELECTRICAL CIRCUITS

Among the main negative effects, we mention: additional losses, overloads, malfunctioning of equipment and protections, the significant shortening of the life span and even the failure of equipment, disruptions of communication networks [20, 27-34].

1.2 STATE OF THE ART

Several solutions have been developed to reduce the presence of harmonics already existing in networks: filters (active, passive or hybrid), oversizing of equipment and conductors, reconfiguration of networks, rewiring, etc. [20, 27-34]. There are also some relatively simple technical solutions for suppressing multiple-of-2 harmonics, as well as multiple-of-3 harmonics.

Computational methods have been developed for solving non-linear circuits in periodic regime using in particular: time domain analysis, frequency domain analysis or combinations between them [21-26].

The time-domain solution does not benefit from single-phase separation. It also cannot take into account different reactances of the synchronous generator for the three sequences. The time-domain solution of electrical circuits with time constants greater than the period can require a very large amount of data to process and a very long computation time. [22, 23, 25, 26]. The Newton-Raphson Method with "shooting" algorithm can be used to speed up the find of the solution [23].

The Harmonic Balance method allows solving the linear part of the circuit by separating the circuit into the 3 sequences, a fact that substantially simplifies the complexity of the three-phase circuit. It results, however, a huge system of nonlinear equations with a large number of unknowns. The nonlinear part is evaluated in the time domain and solved usually using the Newton-Raphson method. There is no certainty of convergence and often under-relaxation must be used. The computational effort is very high. [21-24].

The reduction of the computation effort can be done by using only the first significant harmonics at the terminals of the nonlinear elements and by building models in the frequency domain obtained by measurements of the voltage-current characteristic [21-26]. This way, the computation can be done only in the frequency domain [21-26].

The main advantage of the method is that any kind of nonlinear loads can be modeled using the results of measurements at various frequencies at the terminals of a nonlinear "black box", without taking into account its components.

In order to reduce the computation volume, the number of harmonics considered is usually reduced to a maximum of 25, even with the elimination of harmonics multiples of 2 and 3. In this way, very good calculation speeds can be obtained, but based on rather laborious measurements of characteristics in the frequency domain.

The main disadvantage of the method lies in the difficulty of creating a model close enough to the physical one, as well as obtaining sufficiently accurate results and maintaining an acceptable computation time. Also, the solution using a large number of harmonics is practically impossible. The computation volume and time increase considerably.

An interesting method for solving circuits with non-linear resistive elements, in periodic regime, was proposed by Prof. Hănițilă in [35-38]. The application of the method in solving non-linear three-phase circuits was suggested in: [8, 12, 39], but no numerical examples were presented.

In the present work we analyze the usefulness of applying the Hănițilă method for solving non-linear three-phase circuits and its advantages compared to the main existing methods. We also develop this method through convergence acceleration procedures.

1.3 RESEARCH OBJECTIVES

The main objective of the PhD thesis is the development of the Hănițilă method for solving non-linear three-phase circuits and to perform an analyses of its advantages compared to other existing computation methods. For this we proposed:

- a. To identify the computation algorithm and modeling the Hănițilă method on the computer by developing a computation program. To analyze it in the case of a concrete example of a non-linear three-phase circuit with numerical values.
- b. To analyze how the method behaves in the situation of the existing resonances in the proposed circuit.
- c. To analyze how the Power Balance computed after solving the nonlinear circuit using the Hănițilă method respects the Theory of Power Flow in nonlinear circuits elaborated by acad. A. Țugulea and its further developments and Tellegen's Theorem on Power Conservation.
- d. To use the method for solving three-phase circuits having generators with different reactances on sequences.
- e. To develop and use the method for solving three-phase circuits with non-linear elements with controlled switching (such as thyristors).
- f. To carry out comparative studies with other computation methods.
- g. To analyze the accuracy of the results obtained.
- h. To evaluate the volume and the required computation time, including the case of using a large number of harmonics. To analyze the possibility of using convergence acceleration algorithms as well as to develop such algorithms.
- i. To analyze how the method behaves where sudden variations / jumps of the signal occur.
- j. Analyzing the possibility of avoiding the appearance of the Gibbs Phenomenon.

1.4 PHD THESIS STRUCTURE AND CONTENT

The PHD thesis is divided into 7 chapters, as follows:

Chapter 1 Introduction presents the current trends in electricity consumption and production, the number increase of non-linear elements present in three-phase electrical networks and the problems they can generate. Disturbing effects caused by nonlinear elements are briefly described. An analysis is made of the current state of the art regarding the solutions to mitigate the appearance of harmonics as well as the methods of analyzing non-linear circuits and the existing possibilities of highlighting the disturbing effect. The advantages and disadvantages of these methods are discussed. The main objective of the research is proposed: analyzing the usefulness of the Hăntilă method for solving non-linear three-phase circuits, and the component objectives are established in detail. The structure and the brief content of the thesis are presented by chapters.

Chapter 2 discusses the **Solving of nonlinear three-phase circuits using the Hăntilă Method**. A brief history of the development of the method is presented, with the presentation of bibliographic references and the method uses for various electrical engineering applications. It is analyzed in detail the use of the method for solving of three-phase nonlinear circuits with its two variants: correction in current or correction in voltage of the controlled source. In Illustrative example 1, a three-phase circuit is proposed, which is composed of Boucherot circuits on the 3 phases. It is comparatively solved by using Hăntilă method and LTspice [40] in the time domain. The advantages and disadvantages of the two methods are also compared. The results obtained using the Hăntilă method are validated by those obtained using LTspice [40] in the time domain.

Chapter 3 analyzes the **Solving of nonlinear three-phase circuits having generators with different reactances on sequences** using the Hăntilă method. In illustrative example 2, the three-phase circuit proposed is powered by a generator with apparent poles that presents this particularity. The solution in the time domain cannot take into account the different reactances on the sequences and the possible approximations can lead to results with a rather large error. The circuit analysis being done in the frequency domain, the Hăntilă method allows the computation of elements with different values (reactances or resistances) on harmonics or sequences, an advantage compared to solving in the time domain.

In **Chapter 4**, the **Power flow in non-linear three-phase circuits** resulting from the solution using the Hăntilă method is analyzed. In the illustrative example, the power balances for the circuits solved in Illustrative examples 1 and 2 are computed, verified and confirmed: compliance with the Tellegen theorem and compliance with the acad. A. Țugulea Power flow in three-phase networks with non-linear elements theory and its subsequent developments. The method allows easy tracking of the power flow from the circuit, on harmonics, for each circuit element, including non-linear elements. It also allows to highlight the frequencies at which circuit elements are consuming power and at which they are "generating".

Chapter 5 analyzes the **Solving of three-phase circuits with non-linear controlled switching elements** (such as thyristors) using the Hăntilă method. Since the correction with the nonlinear characteristics is done in the time domain, the method can be easily applied also in the case of nonlinear elements with controlled switching or composed of several characteristics

defined on branches. The method can be applied even when solving in the time domain is not possible due to the need to choose a very small time step. In the illustrative example 3 obtained results are validated by the comparative solution with LTspice [40] in the time domain. It is also proposed and analyzed a solution to avoid the Gibb Phenomenon in the case of solving using the Hăntilă method.

In **Chapter 6**, several **Procedures for accelerating the convergence of the Hăntilă method for solving three-phase circuits with nonlinear elements** are proposed and analyzed. An important advantage of the Hăntilă method is the possibility of using a large number of harmonics, a fact practically impossible in the case of other methods. Depending on the non-linear characteristic and the circuit connected to the terminals of the non-linear element there may be situations where the contraction factor of the algorithm has values very close to 1 and the number of iterations and the computation time may increase substantially. We analyze and develop several procedures for speeding up the calculation algorithm: the optimal choice of the computation resistance R , the use of overrelaxation, the selection of harmonics, the correction of the controlled source in voltage or current. We also propose and detail several original procedures for reducing computational effort and time: the hybrid voltage/current correction procedure, the use of "less harsh" nonlinear characteristics with better contraction factors, the use of modified values for the linear elements of circuit, respectively correcting the non-linear characteristic by including or extracting other existing circuit elements. Illustrative example 4 demonstrates the effectiveness of the proposed acceleration procedures and their practical application.

Chapter 7 presents **Conclusions and original contributions**, as well as prospects for further development. The Hăntilă Method has demonstrated its effectiveness in all analyzed cases and presents several advantages compared to other methods. We believe that the main objective of the doctoral thesis, to develop the method for solving non-linear three-phase circuits and to perform an analysis of its advantages compared to other existing computation methods, has been achieved. Several original contributions have been made to the use of the Hăntilă method both to the solution of non-linear three-phase circuits and to non-linear circuits in general. Several procedures have been developed to accelerate convergence.

We believe that an important development direction of the method is to increase the computation speed. This can be done by identifying more efficient algorithms for computing forward and inverse Fourier transforms (both in speed and accuracy), including non-uniform sampling in the time domain, and by developing the accelerating procedures.

Another interesting direction of development may be to extend the use of the Hăntilă method also for solving circuits with nonlinear inductive elements.

1.5 DISSEMINATION OF THE RESEARCH RESULTS

A part of the results obtained, as well as the analyzes carried out and the conclusions resulting from the present research and which are further presented in the doctoral thesis have been published or are in the process of being published in a series of 4 articles as the first author:

1. **Tufan, C.**; Nemoianu, I.V. Method for the Analysis of Three-Phase Networks Containing Nonlinear Circuit Elements in View of an Efficient Power Flow Computation. *Electronics* **2021**, *10*, 2710.

2. **Tufan, C.**; Nemoianu, I.V.; Maricar, M; Stanculescu, M.; Marin, M. E.; Efficient Method of Harmonic Analysis of Three-Phase Circuits with Nonlinear Controlled Switching Elements, Rev. Roum. Sci. Techn. Électrotechn. et Énerg, vol. 67, 1, pp. 47–54, Bucharest, 2022.

3. **Tufan, C.**; Maricar, M; Nemoianu, I.V.; Procedures for accelerating the convergence of the Hăniță method for solving three-phase circuits with nonlinear elements - Part I, Rev. Roum. Sci. Techn. Électrotechn. et Énerg, vol. 67, 3, pp. 293–300, Bucharest, 2022.

4. **Tufan, C.**; Maricar, M; Nemoianu, I.V.; Procedures for accelerating the convergence of the Hăniță method for solving three-phase circuits with nonlinear elements - Part II - pending publication.

CHAPTER 2. SOLVING OF NONLINEAR THREE-PHASE CIRCUITS USING THE HĂNIȚĂ METHOD

The method has been successfully used in solving several electrotechnical problems with nonlinearities [41, 42]. It is a fixed point method and solves nonlinearity by constructing a convergent Picard-Banach sequence.

For the first time, the method was presented and used in [35], for solving circuits with non-linear resistive elements and then developed in a series of articles [36-38], for periodic circuits. The solution of three-phase circuits using the Hăniță method was proposed in [39] without presenting numerical results.

The use of the method consists in "linearizing" the circuit by replacing the non-linear elements with generators with controlled sources and internal resistances. The value of the sources is iteratively corrected by constructing an algorithm with assured convergence. The value of the internal resistances is chosen to ensure convergence. The correction of controlled sources with non-linear characteristics is done in the time domain. The analysis of linear circuits connected to the terminals of nonlinear elements is done in the frequency domain. Finally, when the value of the controlled source is obtained with a sufficiently good precision, the currents and voltages for all elements in the circuit can be calculated in the frequency domain [36-39]. There are two dual ways of using the method: correction in voltage of the controlled source or correction in current [39].

In the case of voltage correction, the method consists in replacing the non-linear elements with voltage generators composed of controlled voltage sources e and internal resistances R .

The $u - i$ characteristic of the nonlinear element on a phase is described in the time domain by the function $f: \mathbb{R} \rightarrow \mathbb{R}$:

$$i = f(u) \quad (2.1)$$

where i and u are the current and voltage of the nonlinear element, respectively.

$$u = R i + e \quad (2.2)$$

$$e = u - R f(u) = u \left(1 - R \frac{f(u)}{u}\right) = u \left(1 - \frac{R}{R_u(u)}\right) = g(u) \quad (2.3)$$

According to [35, 39] R is chosen so that $g(u)$ is a contraction in the Hilbert space of periodic functions. A sufficient condition for ensuring that $g: \mathbb{R} \rightarrow \mathbb{R}$ is a contraction is that f is a Lipschitz and an uniformly monotone function [37-39, 42].

$$0 < \frac{1}{R_{max}} \stackrel{\text{def}}{=} \lambda \leq \left| \frac{f(u_1) - f(u_2)}{u_1 - u_2} \right| \leq \Lambda \stackrel{\text{def}}{=} \frac{1}{R_{min}} \forall u_1, u_2 \text{ şi } u_1 \neq u_2 \quad (2.7)$$

In order that $g(u)$ is a contraction:

$$\left| 1 - R \frac{f(u_1) - f(u_2)}{u_1 - u_2} \right| = \left| 1 - \frac{R}{\Delta R_{12}} \right| \leq \theta_g < 1 \text{ cu } \Delta R_{12} \in [R_{min}, R_{max}] \quad (2.12)$$

In the case of a non-linear uniport R must be chosen in the range $(0, 2R_{min})$ [39].

The contraction factor of the function $g(u)$ is:

$$\theta_g = \text{Max} \left[\left(1 - \frac{R}{R_{max}} \right), \left(\frac{R}{R_{min}} - 1 \right) \right] \quad (2.13)$$

By replacing the non-linear elements with controlled generators, a linear periodic circuit is obtained. Using harmonic analysis, the three-phase circuit can be decomposed into single-phase DIH sequences. Much simpler linear circuits are obtained in sinusoidal mode. By solving, the voltage at the terminals of the nonlinear element is obtained.

For each harmonic of rank k , the linear circuit connected to the terminals of the nonlinear source can be replaced by the equivalent generator, having source \underline{E}_{gk} and impedance \underline{Z}_{ek} .

Let \underline{U}_k be the voltage at the terminals of the nonlinear element, \underline{I}_k the current through the nonlinear element and \underline{E}_k the voltage of the nonlinearly controlled source, on each harmonic k .

$$\underline{U}_k = (\underline{E}_k + \underline{E}_{gk}) \underline{Z}_{ek} / (\underline{Z}_{ek} + R) = h_k(\underline{E}_k) \quad (2.16)$$

The function h_k is always nonexpansive.

$\underline{U} = h(\underline{E})$, where h is linear diagonal operator with the h_k components. h is nonexpansive.

The iterative solving process from iteration n to iteration $n + 1$:

$$\dots e^{(n)} \xrightarrow{F} \underline{E}^{(n)} \xrightarrow{h} \underline{U}^{(n)} \xrightarrow{F^{-1}} u^{(n)} \xrightarrow{g} e^{(n+1)} \dots \quad (2.20)$$

$e^{(n)}$ is the controlled source of the nonlinear element in the time domain, $\underline{E}^{(n)}$ is the vector of the complex images of the harmonics of the source $e^{(n)}$, $\underline{U}^{(n)}$ is the vector of the complex images of the voltage of at the terminals of the nonlinear element, $u^{(n)}$ is the voltage at the terminals of the nonlinear element in the time domain, F - Fourier transform and F^{-1} - inverse Fourier transform.

We have: the non-expansive function h , the contraction function g and the non-expansive Fourier transform F . The iterative procedure is a composition of nonexpansive functions and a contraction, so it is convergent.

The error (distance) between two successive iteration values at iteration n [39, 41, 42]:

$$\varepsilon^{(n)} \stackrel{\text{def}}{=} \|\underline{E}^{(n)} - \underline{E}^{(n-1)}\| \quad (2.22)$$

Solving the circuit using current correction of the controlled source is dual to voltage correction.

To illustrate the application of the method, we chose a simple circuit in which a synchronous generator, with star connection, feeds a network containing a three-phase rectifier (star with neutral wire). The internal reactances of the generator are equal on sequences to be able to compare the results with those obtained with LTspice [40] in the time domain.

The diodes in the circuit have the piecewise linearized characteristic [36-39].

The computational algorithm was simulated using GNU Octave 6.2.0 [43].

We truncate the Fourier series to the 1000th harmonic and divide the period T using a 8000 equidistant points. We stop the iterations when the distance (error) $\varepsilon^{(n)}$ falls below 10^{-8} .

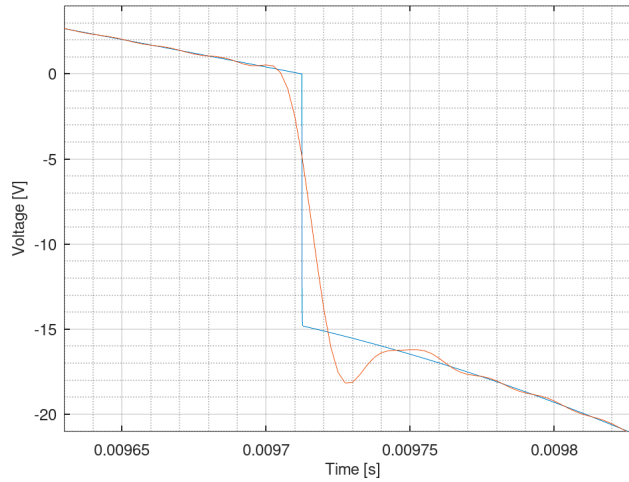


Fig. 2.11 Superimposed time domain graphs for the voltage across the nonlinear element (detail in the Gibbs Phenomenon zone) obtained with: (a) blue - LTspice and (b) red - Hănițilă method

We make the comparison with LTspice in the time domain both by superimposing the graphs obtained in the time domain and by calculating:

$$\varepsilon_{dif} = \sqrt{\frac{\sum \Delta U_n^2 \Delta t_n}{T}} \quad (2.40)$$

where ΔU_n is the difference between the voltage values obtained by LTspice [40] and respectively using the Hănițilă method at point n , and Δt_n is the time step used by LTspice. The maximum value obtained for ΔU_n is 9.6744 V. The big differences are in the area where the Gibbs Phenomenon occurs.

For the norm ε_{dif} we obtained the value 0.15377 V. Even if it is affected by the Gibbs Phenomenon, the accuracy of the results obtained with the Hănițilă method is good enough.

The results are validated by those obtained with LTspice in the time domain.

CHAPTER 3. SOLVING OF NONLINEAR THREE-PHASE CIRCUITS WITH GENERATORS WITH DIFFERENT REACTANCES PER SEQUENCE

The method analyzes the linear circuit connected to the terminals of the non-linear element, in the frequency domain and allows the use of different values of the circuit elements

on sequences or harmonics and is very useful for example in the situation of generators with different reactances on sequences.

The difference appears only when calculating the coefficients $\frac{Z_{e_k}}{Z_{e_k} + R}$ at h , which will be done with the new different values in successions.

In the illustrative example the star-connected synchronous generator has different internal reactances DIH.

The time domain solution cannot take into account the different reactances of the synchronous generator for the three sequences. A possible approximation by using only the

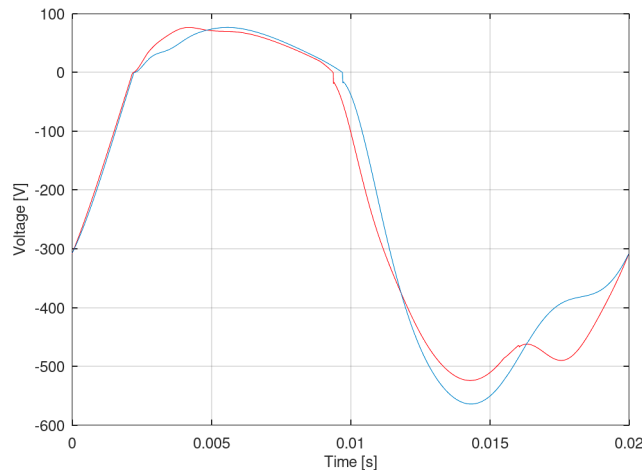


Fig. 3.5 Comparison of the voltage across the nonlinear element in the time domain: blue - Illustrative example 1 and red - Illustrative example 2

value of the reactance on the fundamental for all sequences will not give sufficiently close results.

According to our knowledge, the Hăntilă method is the only method that can be effectively applied to nonlinear circuits having generators with different internal reactances per sequence.

CHAPTER 4. POWER FLOW IN NONLINEAR THREE-PHASE CIRCUITS

Considering the Tellegen theorem that ensures the conservation of powers, the fact that the terms of the Fourier series form an orthonormal basis and the Theory of Power Flow developed by acad. A. Tugulea [1-5] and its further developments [6-13]: The sources of the three-phase generator e_g delivers power in the circuit and also on the internal impedances only on the fundamental frequency. Considering the existence of non-linear elements, it is reinjected on the harmonics. The nonlinear element absorbs power on the fundamental and deliver power on the harmonics through the nonlinearly controlled source. Power balances are verified on harmonics for each frequency.

The Hăntilă method analyzes the circuit in the frequency domain and allows an easy view of the power flow to harmonics. After determining the value of the controlled source, the currents and voltages for all elements in the circuit can be calculated in the frequency domain. Having these values, the harmonic powers can be calculated for all the circuit elements.

We make the power balance for each harmonic for the complex powers for the circuit in Illustrative Example 1 presented in 2.4 for the case of the generator with equal reactances per sequence, respectively for the circuit of Illustrative Example 2 for the case of the generator with different reactances per sequence.

The stated principles are confirmed: the three-phase generator delivers power on the fundamental. Non-linear elements absorb power on the fundamental. Part of this power is consumed internally and the rest is delivered on harmonics. Tellegen's theorem that ensures the conservation of powers is respected.

Although it has an apparently purely resistive characteristic, the non-linear element internally also consumes reactive power.

The validation of the computation is given by the values very close to zero for any of the power balances: on harmonics, on non-linear circuit elements and also on the entire circuit.

A reactance of a lower value by the sequences also has an influence on the fundamental. There is a significant decrease in the reactive power on the generator, accompanied by an increase in the delivered active power.

CHAPTER 5. SOLVING OF THREE-PHASE CIRCUITS WITH NONLINEAR CONTROLLED SWITCHING ELEMENTS

In the case of non-linear elements with controlled switching in the time domain, we will not consider only the dependence of current as a function of voltage, but also as function of time. In fact, there are two characteristics that are switched in the time domain when the command occurs ($t = t_\alpha$) and back when a threshold condition for the current or voltage between the terminals is not any more met. Otherwise, the application of the method is similar to those presented in Chapter 2.

In Illustrative Example 3 we solve a three-phase circuit with thyristors. We use the linearized characteristic [44] for the thyristors.

The rapid variation of the voltage at the thyristor terminals causes the appearance of the Gibbs Phenomenon. We are analyzing a solution to avoid it. We truncate the Fourier series at harmonics of rank 6000 and divide the period T also into a number of 6000 equidistant points. We stop the iterations when the calculated distance (error) $\varepsilon^{(n)}$ drops below 10^{-6} .

Distorted non-sinusoidal behaviors in non-linear three-phase circuits

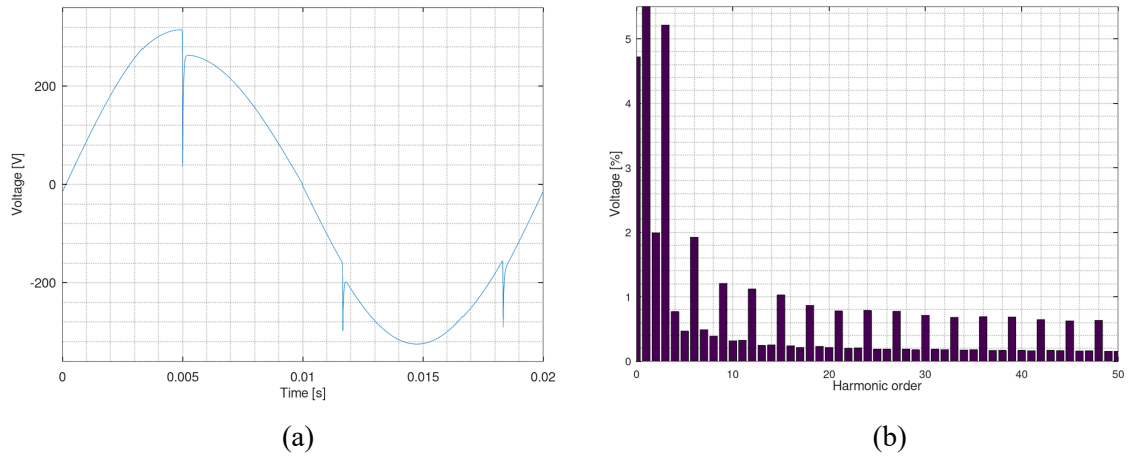


Fig. 5.4 The voltage across the thyristor together with the series resistor: (a) in time domain and (b) harmonic spectrum up to rank 50

In Fig. 5.4a is observed the avoidance of the Gibbs Phenomenon.

The values calculated for the norm ε_{dif} for the voltage on the thyristor and the series resistance for the three simulated variants, compared to the result obtained with LTspice [40] are presented in Tab. 5.1

Table 5.1 Values of the norm ε_{dif} for the voltage on thyristor and the series resistor, for the three simulated variants compared to LTspice.

No. simulation	Harmonic rank	No. sampling points	ε_{dif}
1	6000	6000	2,72 V
2	1000	6000	3,70 V
3	1000	1000	5,75 V

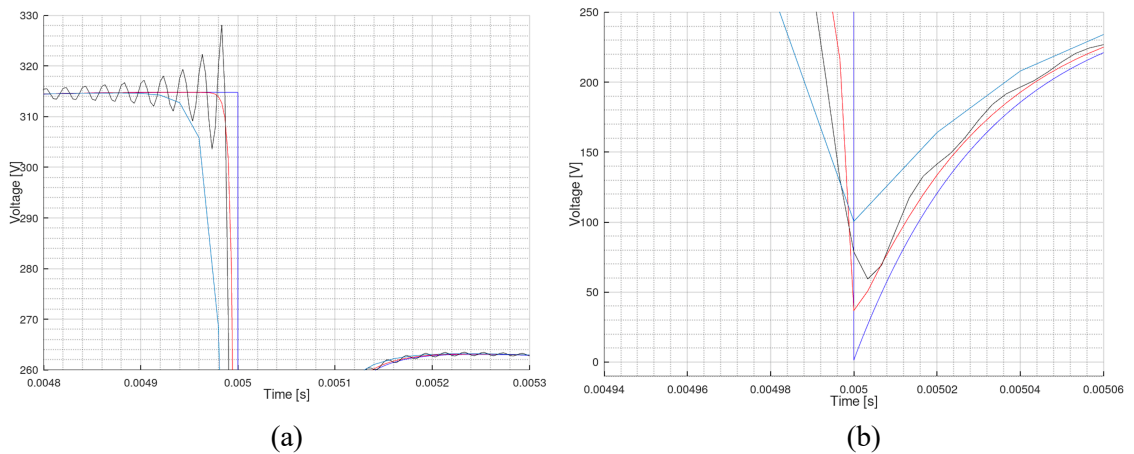


Fig. 5.6 Voltage across the thyristor plus the series resistor by the four simulations: purple - LTspice, red - Simulation 1, black - Simulation 2 and light blue Simulation 3, with details: (a) and (b)

The method can also be easily applied in the case of non-linear elements with controlled switching in the time domain or composed of several characteristics defined on branches. The obtained results are validated by those obtained with LTspice in the time domain. The method can be used even when solving in the time domain is not possible due to the need to choose a very small time step.

In the simulation of circuits using the Hăntilă method, the appearance of the Gibbs Phenomenon can be significantly reduced by keeping a ratio of 1 (+/- 10%) between the number of sampled calculation points and the number of harmonics used to truncate the Fourier series.

CHAPTER 6. PROCEDURES FOR ACCELERATING THE CONVERGENCE OF THE HĂNTILĂ METHOD FOR SOLVING THREE-PHASE CIRCUITS WITH NONLINEAR ELEMENTS

6.1 SPECIFIC CONVERGENCE ACCELERATING PROCEDURES FOR THE PROPOSED METHOD

6.1.1 Optimal choice of computational resistance / conductance

The choice of computation resistance must be made within a certain range to ensure the convergence of the method. The chosen value can also influence the contraction factors and the convergence speed of the algorithm.

In [39] a solution is proposed to minimize the contraction factor θ_g of the function $g(u)$ (defined in subsection 2.2), by choosing

$$R = R_{opt} = \frac{2R_{min}R_{max}}{R_{min} + R_{max}} \quad (6.1)$$

and which leads to the smallest contraction factor for $g(u)$

$$\theta_{g_{opt}} = \frac{R_{max} - R_{min}}{R_{max} + R_{min}} \quad (6.2)$$

It is not mandatory that the convergence speed is higher when we use the optimal resistance R_{opt} . In order to ensure a better convergence speed, one can narrow the selection range of R to $[R_{min}, 2R_{min})$.

By truncating the Fourier series, h is in practice a contraction. For a given circuit and for a finite number of harmonics, a contraction factor can be defined:

$$1 > \theta_h = \max_k \left| 1 / (1 + R / Z_{e_k}) \right| \quad (6.4)$$

A bigger value of R provides a contraction factor θ_h of a smaller value for h , as well as smaller values for the modulus of the coefficients applied to \underline{E}_k .

To ensure the best convergence speed for the iterative procedure the best choice of R should be made in the interval $[R_{min}, 2R_{min})$. Choosing R close to the value R_{opt} is likely to provide very good convergence speed.

The current correction of the controlled source is dual: to ensure the best convergence speed the choice of G must be made in the interval $[G_{min}, 2G_{min})$, with a higher probability close to the G_{opt} value.

6.1.2 Voltage or current correction of the equivalent source

It is possible that one of the two variants (correction in voltage or in current) to provide a shorter calculation time and a smaller number of iterations compared to the other.

If R and G are chosen with: $R = xR_{min}$ and $G = xG_{min}$ with $x \in (0,2)$, then for the same value of x we will have $\theta_g = \theta_{g_i}$.

The evaluation must be done only for θ_h și θ_{h_i} , and respectively the modules of the harmonic correction coefficients of h and h_i .

6.1.3 Over-relaxation

Being based on a convergent Picard Banach sequence with assured convergence, the method allows the use of over-relaxation, which in many situations accelerates convergence [36, 41, 42]:

$$\underline{E}'^{(n)} = \underline{E}^{(n-1)} + \mu(\underline{E}^{(n)} - \underline{E}^{(n-1)}) \text{ cu } \mu > 1 \quad (6.10)$$

Evaluation of the efficiency of over-relaxation applied at iteration n can be done by evaluating the error at iteration $n+1$. The procedure can be used with a fixed value for μ , or a procedure can be developed to find optimal values for μ by testing relation (6.11).

If the contraction factor is appreciably smaller than 1, the use of over-relaxation may not bring a significant improvement.

6.2 FASTER COMPUTATION OF AN INTERMEDIATE RESULT

These acceleration procedures are based on the property of Picard Banach sequences to converge to the solution starting from any initial value and the possibility to calculate in a first phase faster an approximate result close enough to the fixed point and use this result as an input value for the final calculation with the chosen precision and parameters.

6.2.1 Selection of harmonics / use of less accurate calculation algorithms

In [38] is proposed an efficient procedure to reduce the volume and computational time using harmonic selection. As well as a selection algorithm is proposed.

In three-phase electrical networks, the values for the amplitudes of the harmonics are generally decreasing. In this case, a rudimentary harmonic selection method can be used: progressively increasing the number of harmonics and using the result in the next iterative cycle. In most cases, the number of required iterations and the calculation time are decreasing.

Similarly, in a first phase, a smaller number of sampling points can be used, which can be increased in the next phase.

6.2.2 Hybrid voltage / current correction procedure

Starting from the evaluation from 6.1.2, it may be useful to successively use voltage and then current correction of the controlled source (or vice versa).

In the case of inductive circuits, it may be useful to apply the method using voltage correction first by truncating with a smaller number of harmonics. In the second stage, the

number of harmonics is increased and the current correction is used. In such cases a hybrid solution may be faster.

Similarly, in the case of capacitive circuits it may be useful, to apply the method using the current correction and truncating with a smaller number of harmonics. Then the number of harmonics is increased and the voltage correction is used.

The decision to apply the hybrid procedure or direct voltage or current correction can only be made based on the evaluation described in 6.1.2.

6.2.3 The use of linear / nonlinear elements with modified values

A "very hard" nonlinear characteristic can result in contraction factors very close to 1 [42]. In such cases, the circuit can be solved in a first phase using a "less hard" characteristic which ensures a better contraction factor. The result becomes initial value for the computation with the correct characteristic.

By changing the values of certain linear circuit elements, the values for \underline{Z}_{e_k} and \underline{Y}_{e_k} will be changed. These new values influence the contraction factors and coefficients for $h(\underline{E})$ and $h_i(\underline{I}_s)$. An intermediate result can thus be quickly computed.

6.3 CORRECTION OF THE NONLINEAR CHARACTERISTIC BY INCLUDING SOME OTHER EXISTING ELEMENTS FROM THE CIRCUIT

Including an existing series resistor in the nonlinear element conveniently changes the nonlinear $u - i$ characteristic. Initially there is the dependence $i \rightarrow u$ with maximum and minimum slopes bounded by R_{max} și R_{min} . After including the resistor these change to $R_s + R_{max}$ și $R_s + R_{min}$.

$$e = u_2 - R^s f_2(u_2) = g_2(u_2) \quad (6.19)$$

where u_2 is the voltage across the new nonlinear element formed from the original nonlinear element and the series resistance R_s and R^s is the new computation resistance.

The contraction factor of the function $g_2(u_2)$ becomes:

$$\theta_g^s = \text{Max} \left[\left(1 - \frac{R^s}{R_s + R_{max}} \right), \left(\frac{R^s}{R_s + R_{min}} - 1 \right) \right] \quad (6.20)$$

with the selection interval for R^s : $[R_s + R_{min}, 2(R_s + R_{min})]$ (considering also 6.1.1).

A higher value of R_s provides a contraction factor and also better (contraction) coefficients for $g_2(u_2)$.

By including the resistor R_s in the nonlinear element, the equivalent impedance of the circuit connected to the terminals of the new nonlinear element becomes:

$$\underline{Z}_{e_{k2}} = \underline{Z}_{e_k} - R_s \quad (6.21)$$

The coefficient for the new function $h_2(\underline{E})$ for a harmonic of rank k becomes:

$$\left| 1 / (1 + R^s / (\underline{Z}_{e_k} - R_s)) \right| < \left| 1 / (1 + R / \underline{Z}_{e_k}) \right|, \quad (6.23)$$

ensuring also in this case a better contraction factor: $\theta_h^s < \theta_h$.

The inclusion of resistor R_s is good for speeding up the iterative procedure.

Similarly, a parallel resistor can be included in the non-linear characteristic when using current correction.

6.4 CORRECTION OF THE NONLINEAR CHARACTERISTIC BY INCLUDING SOME ELEMENTS TRANSFERED FROM THE EQUIVALENT IMPEDANCE

The possibility to transferr a resistance from the impedance of the circuit connected to the terminals of the nonlinear element allows the generalization of the procedure from 6.3.

The value of the computation resistance must be chosen so that the functions g'_2 and h'_2 to be contractions and ensure a better convergence speed.

For the case of voltage correction, following the transfer of a resistance R'_s from the equivalent impedance Z_{e_k} connected to the terminals of the nonlinear element there will be $\theta'_{g^s} < \theta_g$.

A bigger value of R'_s ensures a better contraction factor for g'_2 and a better convergence speed, as long as the selection interval of R is respected.

For the change of h in h'_2 :

$$\left| \frac{Z_{e_k} - R'_s}{Z_{e_k} + R} \right| < \left| \frac{Z_{e_k}}{Z_{e_k} + R} \right| < 1 \quad (6.24)$$

is satisfied if $R'_s < 2\min_k(\Re(Z_{e_k}))$ with an optimum for $R'_s = \min_k(\Re(Z_{e_k}))$.

Extraction of a parallel resistor using current correction is dual. The equivalent impedance can be brought into the requested form using the series-parallel impedance transformation formulas. One can choose the parallel resistance with the highest value on the harmonics to be extracted.

Similar to 6.1.2, an analysis can be done for the two functions g'_2 și h'_2 , respectively g'_i și h'_i on the contraction factors and the correction coefficients. The faster procedure can be chosen or if necessary one can use the hybrid procedure.

6.5 ILLUSTRATIVE EXAMPLE 4 - ACCELERATION PROCEDURES

We solve a circuit similar to that of 5.3, but with different values of the circuit elements and with a different control angle of the thyristors.

We truncate the Fourier series up to 100th harmonic and divide the period T into 40,000 equidistant points. We stop the iterations when the relative distance (relative error) $\varepsilon^{(n)} / \|e_g\|$ falls below the value of 10^{-8} . Using relative distance allows the comparison of results computed with different metrics.

According to 6.1.2 we analyze the modulus of the harmonic coefficients of h and h_i and the global contraction factors θ_h și θ_{h_i} . From the analysis we have 77 lower values for the coefficients of h and only 24 lower values for h_i - only on the multiple of 3 and high frequency harmonics. Although the global contraction factor is favorable to current correction with $\theta_{h_i} <$

θ_h , due to the distribution of the coefficients modules for h and h_i favorable to voltage correction, we expect that the voltage correction to be faster than the current one.

We also notice the resistor R_s in series with the non-linear element: voltage correction can be used with the inclusion of R_s in the non-linear characteristic.

6.5.1 Voltage correction of the equivalent source

Choosing a higher value for the computation resistance R reduces the time and the number of iterations.

In the case of using overrelaxation, the minimum number of iterations was obtained for $R = 1.5 R_{min}$, being also the fastest result obtained.

For $R = 1.9 R_{min}$ for the case of using overrelaxation there was a decrease in the number of iterations, but a significant increase in the computation time. The economy generated by the use of overrelaxation is in this case smaller than the time required to carry out additional computation.

An overrelaxation factor could not be adopted for $R = R_{opt}$.

6.5.2 Current correction of the equivalent source

Choosing a bigger value for the conductance G reduces the time and the number of iterations.

In the case of using overrelaxation, the fastest solution was obtained for $G = 1.5 G_{min}$.

For the $G = G_{opt}$ case, only an overrelaxation factor of 1.1 could be adopted.

For $G = 1.9 G_{min}$ and $G = G_{opt}$ the use of overrelaxation reduces the number of iterations but significantly increases the computation time.

The decision based on the analysis of the modules of the correction coefficients of the functions h and h_i is confirmed: the voltage correction is in this case faster than the current one.

6.5.3 Correction of the nonlinear characteristic by including of the series resistor R_s

We change the non-linear characteristic by including the series resistor R_s .

A bigger value for the resistance R^s reduces the time and number of iterations. The fastest solution was obtained for the value $R^s = R_{opt}^s$, but also close to it, as in the case of $R^s = 1.9 (R_s + R_{min})$.

The inclusion of the series resistor R_s in the nonlinear characteristic determine a spectacular decrease of the computation times as well as the number of iterations.

The contraction factor being good enough, only small overrelaxation factors can be adopted and the use of overrelaxation does not significantly improve the computation time.

From the analyzed composite acceleration procedures, the fastest proved to be inclusion of the series resistance in the nonlinear characteristic, choose of $R^s = R_{opt}^s$ and use of a computation algorithm with an intermediate step: smaller number of the sampling points.

CHAPTER 7. CONCLUSIONS AND ȘI CONTRIBUȚII ORIGINALE

7.1 GENERAL CONCLUSIONS

The Hăntilă method has demonstrated its effectiveness in all analyzed cases. We note that this has several advantages compared to other methods: ensured convergence, solving on a single phase, the possibility of solving circuits with nonlinear elements with characteristics with controlled switching or defined on branches, the possibility of solving circuits with different values of the circuit elements per harmonics or sequences (for example: generators with different reactances per sequence), the possibility to calculate using a very large number of harmonics.

The method is a useful computation tool that can be easily used starting from the dimensioning and design phase, both of circuits and networks and able to highlight the disturbing effect as well as the power flow on harmonics and to optimally implement and verify possible corrective measures.

The power balance made using the results obtained in the frequency domain by solving the three-phase circuit by the Hăntilă method, respects the Tellegen theorem and confirms the academician A. Țugulea's Theory regarding the power flow in three-phase networks with non-linear elements, and its subsequent developments.

The method brings a spectacular simplification of the computation volume because it allows solving on a single phase. A large part of the computations, for example: the analyze of the circuit connected to the terminals of the nonlinear element or the calculation tables for the inverse Fourier transform must be done only once and are not repeated during the iterations.

Although it is based on complex demonstrations and requires advanced knowledge of functional analysis, the method remains easy to understand, apply and simulate in computer programs.

Convergence is ensured and there is no need to use underrelaxation (as is the case of other methods). In situations with contraction factors close to 1 overrelaxation can be used effectively and the time savings can be significant.

The results obtained with the Hăntilă method are validated by those obtained with LTspice [40] in the time domain. LTspice does not allow single-phase separation for three-phase circuit analysis. The analysis in the time domain avoids the Gibbs Phenomenon that appears in the case of the method presented in this thesis

The method can be used even when solving in the time domain is not possible due to the need to choose a very small time step.

Even if it is affected by the Gibbs Phenomenon (as all methods based on harmonic analysis are), the accuracy of the results is good enough.

The Gibbs Phenomenon can be avoided by keeping a ratio of 1 (+/- 10%) between the number of sampled calculation points and the number of harmonics used to truncate the Fourier series.

The method can be used and is not "blocked" by resonances at different frequencies existing in the analyzed circuit.

If there is needed to consider a large number of harmonics, the volume and time of computation can increase significantly. Depending on the non-linear characteristic and the circuit connected to the terminals of the non-linear element, there may be cases with the contraction factor of the algorithm having values very close to 1. In such cases, acceleration procedures are very useful.

All analyzed acceleration procedures proved to be useful and easy to implement in computing programs. In the cases that the contraction factor is already good enough (significantly lower than 1) or if the truncation is not done at a large number of harmonics, their impact can be a little bit reduced.

Initial evaluation of the harmonic coefficients of h and h_i regarding the decision to use voltage correction, current correction, or hybrid solution is easy to do, and the result was the one expected.

Choosing the computation resistance R close to the R_{opt} value resulted in a substantial reduction in computation time. In the case of a contraction factor very close to 1, an "oscillating convergence" of the error occurred due to computation errors. The phenomenon could be avoided by correcting the non-linear characteristic by including the existing series resistor and using a less "hard" characteristic that ensures a better contraction factor.

In cases with contraction factors close to 1 overrelaxation can be effectively used and the time savings can be significant.

The procedure of modifying the nonlinear characteristic by including existing resistive circuit elements or extracting them from the equivalent impedance connected to the terminals of the nonlinear element has proven to be extremely effective, the effect of reducing the computation time and the number of iterations being spectacular.

We believe that the main objective of the PhD thesis to develop the Hăntilă method for solving non-linear three-phase circuits and to perform an analysis of its advantages compared to other existing computation methods, has been achieved. Several original contributions have been made to the use of the Hăntilă method both to the solution of nonlinear three-phase circuits and to nonlinear circuits in general. Also, the method was developed through convergence acceleration procedures.

7.2 ORIGINAL CONTRIBUTIONS

The main original contributions presented in the PhD thesis are:

1. I identified the computation algorithm of the Hăntilă method and I developed a computation program and used the method in the case of a concrete example of a non-linear three-phase circuit with numerical values. I mention that the method was presented and used in [35], for solving circuits with non-linear resistive elements and then developed in a series of articles [36-38], for circuits in periodic regime. The solution of three-phase circuits using the Hăntilă method was proposed in [39] without presenting numerical results.

2. I analyzed how the method behaves in the case of the resonances in the circuit. The solution using the method was not "blocked" by harmonic resonances present in the proposed circuit containing Boucherot circuits on the 3 phases.

3. I analyzed the way in which the Power Balance computed after solving the non-linear circuit using the Hăntilă method respects the Theory regarding the Power Flow in non-linear circuits elaborated by acad. A. Tugulea and its further developments and Tellegen's Theorem on conservation of powers. The method analyzes the circuit in the frequency domain and allows an easy view of the power flow on harmonics, for each circuit element, including the non-linear elements, and allows the highlighting of the frequencies at which the circuit elements consume power and at which they generate. The power balances respect the Tellegen theorem and confirm the theory of acad. A. Tugulea with its subsequent developments regarding the power flow in three-phase networks with non-linear elements.

4. I proposed and analyzed the use of the Hăntilă method for solving three-phase circuits having generators with different reactances per sequence. Due to the fact that the circuit analysis is done in the frequency domain, the Hăntilă method allows any kind of sequence or harmonic circuit element values and is very useful, for example, for the case of generators with different reactances per sequence.

5. I proposed, developed and analyzed the use of the Hăntilă method for solving three-phase circuits with non-linear elements with controlled switching (such as thyristors). Nonlinear controlled switched resistors are increasingly common in modern equipment. Also, the working principle of active filters is also based on this switching possibility. Since the correction with the nonlinear characteristics is done in the time domain, the method can be easily applied also in the case of nonlinear elements with controlled switching in the time domain or composed of several characteristics defined on the branches.

6. I carried out comparative studies with other computation methods. The results obtained by the Hăntilă method are validated by those obtained with LTspice in the time domain. The method can be used even when solving in the time domain is not possible due to the need to choose a very small time step. LTspice [40] using time domain solving cannot solve circuits with different reactances per sequence and is not useful in such applications. We note that LTspice [40] does not allow single-phase separation for three-phase circuit analysis. But, time domain analysis avoids the Gibbs effect. The method has the possibility to calculate with a very large number of harmonics, a fact practically impossible in the case of other computation methods. The method requires a low computational effort compared to other methods.

7. I carried out an analysis regarding the accuracy of the results obtained. The results obtained by the Hăntilă method are validated by those obtained with LTspice in the time domain. The accuracy of the results can be improved by increasing the number of harmonics and sampling points as well as by choosing a smaller value for the stopping error of the iterations $\varepsilon^{(n)}$.

8. I analyzed the volume and computation time required to use the method in the case of solving three-phase nonlinear circuits and the ability to calculate with a large number of harmonics. The Hăntilă method has the possibility to calculate with a very large number of harmonics, a fact practically impossible in the case of other calculation methods. The method requires a low computational effort compared to other methods.

9. I analyzed the possibility of applying some acceleration algorithms as well as the development of such algorithms. I analyzed, developed and detailed several procedures for accelerating the computation algorithm: the optimal choice of the computation resistance R , the use of overrelaxation, the selection of harmonics, the correction of the controlled source in voltage or in current. I also proposed, developed and detailed several original procedures for reducing computational effort and time: the hybrid voltage/current correction procedure, the use of "less harsh" nonlinear characteristics with better contraction factors, the use of modified values for linear elements of the circuit, respectively correcting the nonlinear characteristic by including other existing circuit elements or extracting them from the equivalent impedance connected to the terminals of the nonlinear element. All analyzed procedures proved to be useful and easy to implement in computation programs.

The procedure of modifying the nonlinear characteristic by including some existing resistive circuit elements proved extremely effective in the analyzed example, the effect of reducing time and the number of iterations being spectacular.

10. I analyzed how the method behaves in the situation where sudden variations of the signal appear. Even if it is affected by the Gibbs Phenomenon, as are all methods based on harmonic analysis, the accuracy of the results obtained by the Hăntilă method is good enough.

11. I analyzed the possibility of avoiding the appearance of the Gibbs Phenomenon in the situation of using the Hăntilă method for solving non-linear three-phase circuits. I proposed, developed and analyzed a solution that consists in keeping a ratio of 1 (+/- 10%) between the number of sampled computation points and the number of harmonics used in Fourier series truncation. The solution avoids the appearance of the Gibbs Phenomenon.

7.3 PROSPECTS FOR FUTURE DEVELOPMENT

We believe that an important direction of development of the Hăntilă method is to increase the computation speed. This can be done by: identifying more efficient algorithms in both speed and accuracy for computing forward and inverse Fourier transforms, including non-uniform sampling in the time domain, and by developing the acceleration procedures.

For the use of overrelaxation, a development direction that we consider useful is the computation of a "dynamic" overrelaxation factor similar to the solution presented in [42] for the electromagnetic field in nonlinear media.

One direction of development of procedures based on faster calculation of an intermediate result is to optimize the way of establishing the intermediate steps and to identify a procedure that allows the evaluation of the step size and the stopping point from the beginning.

It may be interesting to extend the Hăntilă method also for circuits with non-linear inductive elements. In this case, however, it is necessary to develop the solution procedure specific to these circuits. For now, we can specify that the analysis of a circuit containing non-linear resistive and inductive elements is very difficult and can create problems from the point of view of the stability of the solution.

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