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Some classes of operators for nonlinear problems with real world applications

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Abstract of the PhD Thesis

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Contents

Abstract

General objective. This thesis is motivated by the development of fixed point theory in settings where classical assumptions, such as contractiveness or nonexpansiveness, are too restrictive. The work advances this direction by studying generalized metric frameworks and new contractive conditions on the structural side, and by designing efficient, robust iterative schemes suitable for numerical implementation on the algorithmic side. In particular, the thesis unifies several complementary research lines: fixed point results for Istrăţescu contractions in generalized metric spaces, the study of multi-step iterative schemes for generalized nonexpansive mappings, both in W-hyperbolic spaces and in Banach space settings. For these processes, we establish fundamental convergence properties, with particular emphasis on Fejér monotonicity, Δ-convergence, weak and strong convergence. In addition, we propose a two-step inertial algorithm with self-adaptive step sizes for a split common fixed point problem involving demicontractive operators in Hilbert spaces. Finally, the versatility of multi-step processes is illustrated through an application to complex dynamics, where the TTP16-PR structure generates Julia-type and Mandelbrot-type fractals and leads to practical escape criteria for computation.

- *versatility.* Different classes of nonlinear operators are investigated throughout the thesis, including Istrăţescu contractions in extended and new extended b-metric spaces, mappings satisfying Condition (E), L_2 operators, and demicontractive operators. The iterative schemes studied in this work are developed in several ambient frameworks, ranging from Banach and Hilbert spaces to nonlinear geodesic settings such as W-hyperbolic spaces.

- *instrumental value.* The thesis investigates the proposed multi-step iterative algorithms as effective approximation tools for fixed point type problems. Starting from Fejér-type estimates, the analysis yields Δ-convergence and weak convergence in general settings and, under additional mild assumptions on the space or on the operators, strong convergence results suitable for numerical use.

- *qualitative aspects (stability and data dependence).* Designed for finite-precision computation, the presented iterations favor stable behavior and error-control mechanisms (Fejér-type bounds, boundedness arguments, self-adaptive steps) to prevent amplification of rounding errors - thereby supporting reliable numerical outcomes.

Methodology. The methods employed throughout the thesis combine tools from nonlinear analysis with techniques from the theory of iterative algorithms. Fixed point existence and uniqueness results are obtained by adapting contractive-type inequalities to generalized distance settings. The convergence analysis of the studied multi-step schemes relies on Fejér monotonicity, boundedness and asymptotic regularity arguments, together with demiclosedness principles and geometric properties of the underlying spaces, such as uniform convexity in the geodesic setting. Strong convergence results are derived by introducing additional algorithmic control mechanisms, including inertial effects and self-adaptive step sizes, and by applying suitable sequence convergence lemmas. The numerical part of the thesis is supported by explicit escape criteria in the complex plane and by computational experiments implemented in MATLAB.

General state of the art. Fixed point theory provides a fundamental framework for addressing a wide range of problems in nonlinear analysis, optimization, and applied mathematics. Many models can be formulated in terms of the fixed point equation $x = Tx$, where $T\colon X \to X$ is a nonlinear self-mapping acting on a suitable space (X, d). A central objective in this area is to identify general assumptions on the operator and on the underlying space that guarantee the existence (and, when possible, the uniqueness) of fixed points, while retaining enough flexibility to capture phenomena beyond classical contractive settings.

Once existence is ensured, it becomes natural to investigate how such fixed points can be approximated in practice. This motivates the study of iterative procedures, especially multi-step schemes designed for broad classes of generalized nonexpansive-type mappings. The convergence analysis becomes more delicate beyond linear structures, since many classical arguments rely on properties specific to normed spaces. Nevertheless, several nonlinear geometric settings retain enough convexity to support meaningful convergence theory and allow the extension of iterative methods to spaces without linear structure.

In this thesis, these two directions are developed in parallel. On the one hand, we establish fixed point results for Istrăţescu contractions in generalized distance frameworks, such as extended b-metric spaces. On the other hand, we investigate multi-step iteration schemes in nonlinear geodesic settings, with particular emphasis on W-hyperbolic spaces, where notions such as Fejér monotonicity, Δ-convergence and strong convergence can be studied in a unified way. Moreover, we develop and analyze iterative procedures in Banach and Hilbert spaces, including the TTP16-PR iteration for L_2 operators and a two-step inertial algorithm with self-adaptive step sizes for split common fixed point problems involving demicontractive mappings. Finally, the versatility of the multi-step framework is illustrated through an application to complex dynamics, where the TTP16-PR iteration is employed as an alternative orbit rule for generating Julia-type and Mandelbrot-type fractals.

Thesis description: structure and content.

Chapter 1, **Istrăţescu contractions on extended b-metric spaces** ([31], [29], [30]), is devoted to the study of several classes of Istrăţescu contractions in the setting of extended b-metric spaces and new extended b-metric spaces. The main purpose of this chapter is to establish existence and uniqueness results for fixed points of such mappings, by combining assumptions such as orbital continuity with explicit conditions on the control function θ.

The starting point of the chapter is the framework of extended b-metric spaces, introduced by Kamran *et al.* [14], where the classical triangle inequality is replaced by a weakened inequality involving a control function $\theta\colon X \times X \to [1, \infty)$.

Definition 1 ([14])**.** Let X be a nonempty set and let $\theta\colon X \times X \to [1, \infty)$. A function $d\colon X \times X \to [0, \infty)$ is said to be an extended b-metric if, for all x, y, $z \in X$, the following conditions are satisfied:

(i) $d(x, y) = 0$ if and only if $x = y$;

(ii) $d(x, y) = d(y, x)$;

(iii) $d(x, z) \le \theta(x, z)\,\big[d(x, y) + d(y, z)\big]$.

The pair (X, d) is called an extended b-metric space.

The core of the chapter is represented by several classes of Istrăţescu contractions, including Istrăţescu contractions of order 2, two-sided Istrăţescu contractions, Istrăţescu contractions of type 2, Istrăţescu contractions of order k, and a new class introduced here, namely the Ćirić-Istrăţescu contractions.

Definition 2. A mapping $T\colon X \to X$ is said to be a Ćirić-Istrăţescu contraction if there exists $h \in [0, 1)$ such that, for all x, $y \in X$, the following inequality holds:

$$d(T^2x, T^2y) \le h\,\max\{d(x, y), d(Tx, Ty), d(x, Tx), d(Tx, T^2x), d(y, Ty), d(Ty, T^2y)\}.$$

This class extends classical Ćirić contractions, Istrăţescu contractions of order 2, two-sided Istrăţescu contractions and Istrăţescu contractions of type 2.

A key role in the proofs is played by orbital continuity, which replaces continuity in the transition from convergence of Picard iterations to existence of fixed points.

Definition 3 **([6]).** A self-mapping $T\colon X \to X$ is said to be orbitally continuous if $\lim_{n\to\infty} T^n x = u$ implies $\lim_{n\to\infty} T(T^n x) = Tu$, where T^n denotes the n-fold composition of T with itself.

The main technical tools of the chapter are the Cauchy criteria for Picard iteration sequences in extended b-metric spaces, formulated in Lemma 1, Lemma 2. These results rely on explicit estimates for successive distances between iterates and on product-type assumptions involving the control function θ.

Lemma 1. *Let (X, d) be an extended b-metric space, let $T\colon X \to X$ be a mapping and $\{x_n\}$ be the sequence of Picard iterations generated by an initial point $x_0 \in X$, that is, $x_n = T^n x_0$, for all $n \geq 0$. If there exists $\lambda \in [0, 1)$ such that the inequality $d(x_n, x_{n+1}) \leq \sqrt{\lambda^{n-1}} \max\{d(x_0, x_1), d(x_1, x_2)\}$ holds for all $n \geq 0$, and there exist constants $\beta < \frac{1}{\sqrt{\lambda}}$ and $n_0 \geq 0$ such that the inequality $\prod_{i=n}^{j} \theta(x_i, x_{n+p}) \leq \beta^{j-n+1}$ holds for all $n \geq n_0$, $p \geq 1$ and $j \in \{n, n+1, \dots, n+p-1\}$, then $\{x_n\}$ is a Cauchy sequence.*

Lemma 2. *Let (X, d) be an extended b-metric space, let $T\colon X \to X$ be a mapping, and let $\{x_n\}$ be the Picard iteration sequence based on an initial point $x_0 \in X$. If there exist $\lambda \in [0, 1)$ and an integer $k \geq 2$ such that $d(x_n, x_{n+1}) \leq \sqrt[k]{\lambda^{n-k}}\, M$, for all $n \geq 0$, where $M = \max\{d(x_0, x_1), d(x_1, x_2), \dots, d(x_{k-1}, x_k)\}$ and there exist $\beta < \frac{1}{\sqrt[k]{\lambda}}$ and $n_0 \geq 0$ such that the inequality $\prod_{i=n}^{j} \theta(x_i, x_{n+p}) \leq \beta^{j-n+1}$ holds for all $n \geq n_0$, $p \geq 1$ and $j \in \{n, n+1, \dots, n+p-1\}$, then $\{x_n\}$ is a Cauchy sequence.*

Based on these results, the first part of the chapter establishes fixed point theorems in complete extended b-metric spaces for the classes of mappings introduced above.

Theorem 1. *Let (X, d) be a complete extended b-metric space and let $T\colon X \to X$ be an orbitally continuous Istrăţescu contraction of order 2, and fix $x_0 \in X$. Set $x_n = T^n x_0$ for all $n \geq 0$. Assume that there exist $\beta < \frac{1}{\sqrt{a+b}}$ and $n_0 \geq 0$ such that the inequality $\prod_{i=n}^{j} \theta(x_i, x_{n+p}) \leq \beta^{j-n+1}$ holds for all $n \geq n_0$, $p \geq 1$ and $j \in \{n, n+1, \dots, n+p-1\}$. Then T has a unique fixed point.*

Theorem 2. *Let (X, d) be a complete extended b-metric space and let $T\colon X \to X$ be an orbitally continuous Ćirić-Istrăţescu contraction. Let $x_0 \in X$ and denote by $\{x_n\}$ the associated Picard sequence. Presume there exist $\beta < \frac{1}{\sqrt{h}}$ and $n_0 \geq 0$ such that the* *inequality* $\prod_{i=n}^{j}\theta(x_i,x_{n+p})\leq\beta^{j-n+1}$ *holds for all* $n\geq n_0$*,* $p\geq 1$ *and* $j\in\{n,n+1,\dots,n+p-1\}$*. Then* T *has a unique fixed point.*

Theorem 3. *Let* (X,d) *be a complete extended* b*-metric space and let* k *be an integer such that* $k\geq 2$*, and let* $T\colon X\to X$ *be an orbitally continuous Istrăţescu contraction of order* k*. Fix* $x_0\in X$ *and consider* $x_n=T^nx_0$ *for all* $n\geq 0$*. Suppose there exist* $\beta<\frac{1}{\sqrt[k]{a_0+a_1+\cdots+a_{k-1}}}$*, and* $n_0\geq 0$ *such that the inequality* $\prod_{i=n}^{j}\theta(x_i,x_{n+p})\leq\beta^{j-n+1}$ *holds for all* $n\geq n_0$*,* $p\geq 1$ *and* $j\in\{n,n+1,\dots,n+p-1\}$*. Then* T *has a unique fixed point.*

The applicability of these results is illustrated by an example showing that the assumptions of Theorem 1 are fulfilled by an operator acting on an extended b-metric space which is not a b-metric space, and whose unique fixed point is 0.

The second part of the chapter extends the analysis to the broader class of new extended b-metric spaces, introduced by Aydi *et al.* [2], where the control function depends on three variables.

Definition 4 ([2])**.** Let X be a nonempty set and $\theta\colon X\times X\times X\to[1,\infty)$. A new extended b-metric is a function $d\colon X\times X\to[0,\infty)$ such that for all x, y, $z\in X$,

(i) $d(x,y)=0$ if and only if $x=y$,

(ii) $d(x,y)=d(y,x)$,

(iii) $d(x,z)\leq\theta(x,y,z)\,[d(x,y)+d(y,z)]$.

The pair (X,d) is called a new extended b-metric space.

For this framework, the chapter derives analogous upper bounds for successive Picard distances for several classes of contractions, and then establishes Cauchy criteria together with fixed point results under suitable assumptions on the three-variable control function.

The chapter then presents analogous fixed point results corresponding to these classes of contractions, extending the conclusions obtained in the previous framework.

Thus, the chapter establishes a unified fixed point theory for several classes of Istrăţescu contractions in both extended b-metric spaces and new extended b-metric spaces, showing in particular that orbital continuity together with product-type assumptions on the control function θ are sufficient to guarantee existence and uniqueness of fixed points in these generalized settings.

Chapter 2, **Operators with Condition** (E) **in** W**-hyperbolic spaces** ([31], [33]), is devoted to the study of mappings satisfying Condition (E) in the nonlinear framework of W-hyperbolic spaces. The main objective is to establish convergence results for the SRJ iteration process in this setting, including Fejér monotonicity, Δ-convergence and strong convergence. In addition, the chapter contains a nonlinear integral example showing that the theory applies to operators satisfying Condition (E) which are not nonexpansive.

The chapter begins with the class of mappings satisfying Condition (E), introduced by García-Falset *et al.* [12], which properly contains the class of Suzuki mappings satisfying Condition (C). Within this framework, Condition (E) yields a class of operators that is broad enough to go beyond nonexpansive mappings, while still allowing an effective fixed point approximation theory. Also, it is shown that every mapping satisfying Condition (E) and admitting a fixed point is quasi-nonexpansive.

Definition 5 ([12])**.** Let (X, d) be a metric space, let C be a nonempty subset of X and $\mu \geq 1$. A mapping $T\colon C \to C$ satisfies Condition (E_μ) on the set C if

$$d(x, Ty) \leq \mu\, d(x, Tx) + d(x, y),$$

for all x, $y \in C$.

Moreover, the mapping T is said to satisfy Condition (E) if there exists a constant $\mu \geq 1$ for which T satisfies Condition (E_μ).

The ambient framework of the chapter is that of W-hyperbolic spaces, introduced by Kohlenbach [16], which provides a nonlinear metric setting equipped with a convexity structure.

Definition 6 ([16])**.** A metric space (X, d) is said to be a W-hyperbolic space if there exists a mapping $W\colon X^2 \times [0,1] \to X$ satisfying the following conditions for all x, y, z, $w \in X$ and α, $\beta \in [0,1]$:

(i) $d(z, W(x, y, \alpha)) \leq (1-\alpha)\, d(z, x) + \alpha\, d(z, y)$,

(ii) $d(W(x, y, \alpha), W(x, y, \beta)) = |\alpha - \beta|\, d(x, y)$,

(iii) $W(x, y, \alpha) = W(y, x, 1-\alpha)$,

(iv) $d(W(x, z, \alpha), W(y, w, \alpha)) \leq (1-\alpha)\, d(x, y) + \alpha\, d(z, w)$.

A substantial part of the preliminary section is devoted to asymptotic centers and Δ-convergence in uniformly convex W-hyperbolic spaces. In this setting, we rely on the uniqueness of asymptotic centers for bounded sequences and on some technical results describing the asymptotic behaviour of sequences in the space. These are used in the convergence analysis later in the chapter.

The central iterative procedure considered in this chapter is the SRJ process, originally introduced by Dashputre *et al.* [11], here adapted to the setting of W-hyperbolic spaces.

Algorithm 1 SRJ-iteration in W-hyperbolic spaces

Let C be a nonempty, closed, and convex subset of a W-hyperbolic space (X, d, W) and $T\colon C \to C$ a mapping. Given an arbitrary initial point $x_1 \in C$, the sequence $\{x_n\}$ is generated by

$$\begin{cases} z_n = T(W(x_n, Tx_n, \alpha_n)), \\ y_n = T(W(z_n, Tz_n, \beta_n)), \\ x_{n+1} = T(W(y_n, Ty_n, \gamma_n)), \quad n \geq 1 \end{cases}$$

where $\{\alpha_n\}$, $\{\beta_n\}$ and $\{\gamma_n\}$ are control sequences in $(0, 1)$.

The first important property of the iterative sequence is established in Theorem 4, which proves that the SRJ iteration is Fejér monotone with respect to the fixed point set.

Theorem 4. *Let C be a nonempty, closed, and convex subset of a W-hyperbolic space (X, d, W). Let $T\colon C \to C$ be a mapping satisfying Condition* (E)*, with* $\mathrm{Fix}(T) \neq \emptyset$*. If $\{x_n\}$ is a sequence defined by Algorithm* 1*, then $\{x_n\}$ is Fejér monotone with respect to* $\mathrm{Fix}(T)$*.*

A characterization result is then obtained in Theorem 5, which connects the existence of fixed points with boundedness and asymptotic regularity of the generated sequence.

Theorem 5. *Let C be a nonempty, closed, and convex subset of a complete uniformly convex W-hyperbolic space (X, d, W) with a monotone modulus of uniform convexity and let $T\colon C \to C$ be a mapping satisfying Condition* (E)*. If $\{x_n\}$ is a sequence defined by Algorithm* 1 *with $\alpha_n \in [a, b] \subset (0, 1)$, then* $\mathrm{Fix}(T)$ *is nonempty if and only if the sequence $\{x_n\}$ is bounded and* $\lim_{n\to\infty} d(x_n, Tx_n) = 0$*.*

One of the main convergence result of the chapter is the following Δ-convergence theorem.

Theorem 6. *Let C be a nonempty, closed, and convex subset of a complete uniformly convex W-hyperbolic space (X, d, W) with a monotone modulus of uniform convexity. Let $T\colon C \to C$ be a mapping satisfying Condition* (E)*, with* $\mathrm{Fix}(T) \neq \emptyset$*. Then the sequence $\{x_n\}$ defined by Algorithm* 1 *is Δ-convergent to a fixed point of T.*

The convergence analysis is completed by two additional results regarding strong convergence. The first one characterizes strong convergence in terms of the asymptotic distance to the fixed point set.

Theorem 7. *Let C be a nonempty, closed, and convex subset of a complete uniformly convex W-hyperbolic space (X, d, W) with a monotone modulus of uniform convexity and let $T\colon C \to C$ be a mapping satisfying Condition* (E)*, with* $\mathrm{Fix}(T) \neq \emptyset$*. Then the sequence $\{x_n\}$ defined by Algorithm* 1 *converges strongly to some fixed point of T if and only if* $\liminf_{n\to\infty} D(x_n, \mathrm{Fix}(T)) = 0$*, where* $D(x, A) = \inf\{d(x, y) : y \in A\}$*.*

The second one shows that Condition (I) yields a direct sufficient criterion for strong convergence.

Definition 7 ([32])**.** A mapping T defined on a subset C of a W-hyperbolic space (X, d, W) with a nonempty fixed point set $\mathrm{Fix}(T)$ is said to satisfy Condition (I) if there exists a nondecreasing function $f\colon [0, \infty) \to [0, \infty)$ such that $f(0) = 0$ and $f(t) > 0$ for every $t > 0$, and the inequality

$$d(x, Tx) \geq f\big(D(x, \mathrm{Fix}(T))\big)$$

holds for all $x \in C$.

Theorem 8. *Let C be a nonempty, closed, and convex subset of a complete uniformly convex W-hyperbolic space (X, d, W) with a monotone modulus of uniform convexity and let $T\colon C \to C$ be a mapping satisfying Condition* (E)*. Moreover, T satisfies Condition* (I) *with* $\mathrm{Fix}(T) \neq \emptyset$*. Then the sequence $\{x_n\}$ defined by Algorithm* 1 *converges strongly to a fixed point of T.*

The numerical part of the chapter first includes a comparison between the SRJ process and four representative multi-step iterations from the literature, namely the Abbas-Nazir [1], Thakur *et al.* [35], Piri *et al.* [26], and Kumar-Pathak [17] iterative schemes. This comparison is carried out for a mapping on $[0, 1]$ satisfying Condition (E) but not Condition (C), thus showing that the proposed framework goes genuinely beyond the classical Suzuki setting.

The numerical data and graphs obtained for several parameter configurations show that all considered schemes converge, while the SRJ process reaches the fixed point faster than the competing methods in the tested cases.

An important contribution of this chapter is a novel nonlinear integral example in $L^2([0, 1])$, introduced in this work, which illustrates that the theory applies to operators satisfying Condition (E) without being nonexpansive. This example extends the scope of the chapter beyond finite-dimensional and demonstrates the applicability of the SRJ process in a genuinely nonlinear functional setting.

Example 1. Let $\theta\colon [0,1]\times[0,1]\to\mathbb{R}$ be a continuous function satisfying $\theta(t,s)\in[0,1]$ for all t, $s\in[0,1]$. Consider the Banach space $X=L^2([0,1])$ endowed with the usual L^2-norm

$$\|u-v\|=\left(\int_0^1 |u(t)-v(t)|^2\,dt\right)^{1/2}.$$

We define the operator

$$\mathcal{T}\colon X\to X,\qquad (\mathcal{T}u)(t)=\begin{cases}\dfrac{1}{2}\left(\displaystyle\int_0^1 \theta(t,s)\,u^2(s)\,ds\right)^{1/2}, & \text{if } u\not\equiv 1,\\ -\dfrac{1}{2}, & \text{if } u\equiv 1,\end{cases}$$

where $u\equiv 1$ denotes the function equal to 1 almost everywhere on $[0,1]$.

For this operator, it is shown that $\mathcal{T}$ satisfies Condition (E_3), while it is not nonexpansive. Moreover, numerical experiments based on the kernel

$$\theta(t,s)=\frac{1+\cos(\pi ts)}{2},\quad t,s\in[0,1],$$

and several initial profiles show that the SRJ scheme converges strongly to the fixed point $u\equiv 0$ within a small number of iterations. This example confirms the practical effectiveness of the SRJ process in a nonlinear framework and represents an essential application of the theoretical results established in the chapter.

Chapter 3, L_2 **operators with common fixed point via new iteration** ([31], [27]), is devoted to the approximation of common fixed points of two L_2 operators in the setting of uniformly convex Banach spaces. The class of L_2 operators was derived by Latif, Postolache and Alansari [18] by isolating the second part of the Condition (L) introduced by Llorens-Fuster and Moreno-Gálvez [20]. More precisely, if C is a nonempty, bounded and convex subset of a uniformly convex Banach space $(X,\|\cdot\|)$, a mapping $T\colon C\to C$ is called an L_2 operator if, for every almost fixed point sequence $\{x_n\}$ of T in C and for every $x\in C$, one has

$$\limsup_{n\to\infty}\|x_n-Tx\|\le\limsup_{n\to\infty}\|x_n-x\|.$$

The main objective of this chapter is to study a common fixed point problem for two operators $T\colon C\to C$ and $S\colon C\to C$ belonging to this class. In order to do so, a new iterative algorithm is introduced, inspired by the TTP16 iteration of Thakur *et al.* [35], but adapted to the two-operator framework. The proposed scheme, called the TTP16-PR iteration, preserves the nested structure of the original procedure while incorporating the second operator at the intermediate step.

Algorithm 2 TTP16-PR algorithm

Let C be a nonempty, closed, and convex subset of a Banach space $(X, \|\cdot\|)$. For two mappings $T\colon C \to C$ and $S\colon C \to C$ and $x_1 \in C$, the sequence $\{x_n\}$ is generated in three steps by the rules:

$$\begin{cases} z_n = (1-\beta_n)x_n + \beta_n T x_n, \\ y_n = T((1-\alpha_n)Sx_n + \alpha_n z_n), \\ x_{n+1} = Ty_n, \end{cases}$$

for all $n \geq 1$, where $\{\alpha_n\}$ and $\{\beta_n\}$ are real sequences in $(0,1)$.

A first important step in the analysis consists in proving that the generated iterates are Fejér monotone with respect to the set $\mathrm{Fix}(T) \cap \mathrm{Fix}(S)$, and that several associated sequences converge to the same limit. These properties are established in Lemma 3.

Lemma 3. *Let C be a nonempty, closed, and convex subset of a Banach space $(X, \|\cdot\|)$, $T\colon C \to C$, $S\colon C \to C$ be quasi-nonexpansive mappings, and $0 < A \leq \alpha_n, \beta_n \leq B < 1$, for any $n \geq 1$. If T and S have a common fixed point, then:*

1. *$\{x_n\}$ and $\{y_n\}$ are Fejér monotone sequences with respect to $\mathrm{Fix}(T) \cap \mathrm{Fix}(S)$;*
2. *For any fixed point $p \in \mathrm{Fix}(T) \cap \mathrm{Fix}(S)$, the sequences $\{\|z_n - p\|_n\}$, $\{\|Tx_n - p\|_n\}$ and $\{\|Sx_n - p\|_n\}$ converge to the same real number.*

Based on these preparatory facts, Theorem 9 provides a characterization of the common fixed point problem in terms of boundedness and asymptotic regularity of the generated sequence.

Theorem 9. *Let C be a nonempty, closed, and convex subset of a uniformly convex Banach space $(X, \|\cdot\|)$, and let $T\colon C \to C$, $S\colon C \to C$ be two operators. For arbitrary chosen $x_1 \in C$, let the sequence $\{x_n\}$ be generated by the process defined by Algorithm 2, where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of real numbers in $[a,b]$ for some a, b with $0 < a \leq b < 1$. Then the next statements hold true:*

1. *If S and T are quasi-nonexpansive and $\mathrm{Fix}(T) \cap \mathrm{Fix}(S) \neq \emptyset$, then $\{x_n\}$ is bounded and $\lim_{n\to\infty} \|x_n - Tx_n\| = \lim_{n\to\infty} \|x_n - Sx_n\| = 0$.*
2. *If $\{x_n\}$ is bounded, $\lim_{n\to\infty} \|x_n - Tx_n\| = \lim_{n\to\infty} \|x_n - Sx_n\| = 0$ and T and S are L_2 operators, then $\mathrm{Fix}(T) \cap \mathrm{Fix}(S) \neq \emptyset$.*

The weak convergence analysis is then carried out under the additional assumption that the ambient uniformly convex Banach space has the Opial property. In this direction, Lemma 4 gives a demiclosedness-type result for two L_2 operators.

Lemma 4. *Let $T\colon C \to C$, $S\colon C \to C$ be L_2 operators on a subset C of a Banach space $(X, \|\cdot\|)$ that satisfies the Opial property. If $\{x_n\} \subset C$ converges weakly to $z \in C$ and $\lim_{n\to\infty} \|Tx_n - x_n\| = \lim_{n\to\infty} \|Sx_n - x_n\| = 0$, then z is a common fixed point of T and S, i.e. $Tz = Sz = z$.*

By combining this result with the boundedness and asymptotic regularity obtained earlier, the main weak convergence theorem is established.

Theorem 10. *In the setting of a uniformly convex Banach space $(X, \|\cdot\|)$ with the Opial property, let $T\colon C \to C$, $S\colon C \to C$ be two L_2 operators defined on a nonempty, closed, and convex subset C of X. Assume that T and S have at least one common fixed point. Then, for any initial point $x_1 \in C$, the sequence $\{x_n\}$ generated by Algorithm 2 converges weakly to a point $z \in \mathrm{Fix}(T) \cap \mathrm{Fix}(S)$.*

The chapter also contains two strong convergence results. The first one is obtained under compactness assumptions on the set C.

Theorem 11. *Let C be a compact, nonempty, and convex subset of a uniformly convex Banach space $(X, \|\cdot\|)$ and let T, $S\colon X \to X$ be L_2 operators with $\mathrm{Fix}(T) \cap \mathrm{Fix}(S) \neq \emptyset$. For arbitrary chosen $x_1 \in C$, let the sequence $\{x_n\}$ be generated by Algorithm 2, where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of real numbers in $[a, b]$ for some a, b with $0 < a \leq b < 1$. Then $\{x_n\}$ converges strongly to a common fixed point of T and S.*

A second route to strong convergence is obtained by using Condition (I), without requiring compactness. In this setting, Proposition 1, Lemma 5 and Theorem 12 provide the necessary arguments.

Proposition 1. *If T is an L_2 operator, then $\mathrm{Fix}(T)$ is a closed set.*

Lemma 5. *Let C be a closed, nonempty, and convex subset of a uniformly convex Banach space $(X, \|\cdot\|)$ and let T, $S\colon C \to C$ be L_2 operators with $\mathrm{Fix}(T) \cap \mathrm{Fix}(S) \neq \emptyset$. If the sequence $\{x_n\}$ generated by Algorithm 2 satisfies the equalities*

$$\liminf_{n\to\infty} \|x_n - \mathrm{Fix}(T)\| = \liminf_{n\to\infty} \|x_n - \mathrm{Fix}(S)\| = 0,$$

then $\{x_n\}$ is strongly convergent to a common fixed point of T and S.

Theorem 12. *Let C be a closed, nonempty, and convex subset of a uniformly convex Banach space $(X, \|\cdot\|)$ and let T, $S\colon X \to X$ be L_2 operators with $\mathrm{Fix}(T) \cap \mathrm{Fix}(S) \neq \emptyset$. For arbitrary chosen $x_1 \in C$, let the sequence $\{x_n\}$ be generated by Algorithm 2, where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences of real numbers in $[a, b]$ for some a, b with $0 < a \leq b < 1$.*

If T and S satisfy Condition (I)*, then $\{x_n\}$ converges strongly to a common fixed point of T and S.*

The last part of the chapter is devoted to a numerical comparison between the proposed TTP16-PR iteration and several classical and recent two-operator schemes, namely the Das-Debata process [10], the iteration of Liu *et al.* [19], the Mann-type process of Khan *et al.* [15], the Y-iteration introduced by Yadav [36], and the N-iteration proposed by Navascués [24].

To perform the comparison, in this chapter it is used the nonlinear integral-type operator introduced in the previous chapter, consisting of two concrete nonlinear L_2 operators defined on $L^2([0,1])$ via suitable kernel constructions:

$$\theta_1(t,s) = \frac{1+\cos\big(\pi(t-s)\big)}{2} \quad \text{and} \quad \theta_2(t,s) = e^{-20(t-s)^2}.$$

These kernels generate the operators $T\colon L^2([0,1]) \to L^2([0,1])$, $S\colon L^2([0,1]) \to L^2([0,1])$, which have the common fixed point $u \equiv 0$.

The numerical results obtained for this benchmark show that all considered schemes converge, while the proposed TTP16-PR method reaches the common fixed point in fewer iterations and requires less computational time than the competing methods in the tested cases.

Chapter 4, **Split feasibility problem with demicontractive operators** ([31], [21]), is devoted to the study of the split common fixed point problem in real Hilbert spaces for demicontractive operators. The main contribution of this chapter is the introduction of a new two-step inertial algorithm with self-adaptive step sizes for solving the split common fixed point problem, together with a strong convergence theorem toward the minimum norm solution. In addition, the proposed approach combines inertial effects with adaptive control in a unified framework, allowing both acceleration and stability of the iterative process. An important advantage of the method is that the convergence analysis does not require the explicit computation of the norm of the bounded linear operator, which is often difficult to estimate or even unavailable in practical applications.

The chapter starts with the general split feasibility problem, which consists of finding a point

$$x \in C \text{ such that } Ax \in Q,$$

where C and Q are nonempty, closed and convex subsets of Hilbert spaces H_1 and H_2, and $A\colon H_1 \to H_2$ is a bounded linear operator. This formulation naturally arises in applications where constraints are imposed in different spaces linked by a linear transformation, such as signal processing, image reconstruction or inverse problems. When the sets C and Q are represented as fixed point sets of nonlinear operators, one arrives at the split common fixed point problem (SCFP), namely to find

$$x \in \mathrm{Fix}(S) \text{ such that } Ax \in \mathrm{Fix}(T),$$

where $S\colon H_1 \to H_1$ and $T\colon H_2 \to H_2$ are given operators. The corresponding solution set is

$$\Omega = \{x \in H_1 : x \in \mathrm{Fix}(S) \text{ and } Ax \in \mathrm{Fix}(T)\}.$$

This formulation provides a natural bridge between feasibility problems and fixed point theory in a coupled setting.

In order to formulate the problem precisely and to develop a convergence theory, the chapter recalls the class of demicontractive mappings, which plays a central role in the analysis. More precisely, if $T\colon H \to H$ is an operator with nonempty fixed point set, then T is called β-demicontractive, with $0 \le \beta < 1$, if

$$\|Tx - u\|^2 \le \|x - u\|^2 + \beta \|x - Tx\|^2,$$

for all $u \in \mathrm{Fix}(T)$ and $x \in H$. This class properly extends several well-known classes of nonlinear operators, including nonexpansive-type mappings, and provides a flexible framework for the study of split problems in Hilbert spaces.

The chapter also discusses earlier approaches to the split feasibility and split common fixed point problems, starting with Byrne's classical CQ algorithm [4], continuing with extensions due to Censor and Segal [5], Moudafi [23], Cui and Wang [7], Boikanyo [3], Dang *et al.* [9], Cui *et al.* [8], and Suparatulatorn *et al.* [34]. These methods progressively incorporate more general classes of operators and improved convergence mechanisms. However, a common difficulty in many of these approaches is the dependence of the step-size choice on the norm of the operator or related quantities, which may be inconvenient or impractical in applications. This motivates the search for self-adaptive schemes that avoid such requirements. At the same time, inertial methods inspired by Polyak [28] and Nesterov [25] suggest that acceleration can be achieved by incorporating information from previous iterates into the update rule, leading to faster convergence in practice.

Motivated by these developments, in this chapter it is introduced a new two-step inertial algorithm for the SCFP, combining self-adaptive step sizes with a two-step inertial term. The proposed method balances stability and acceleration, while ensuring strong convergence to the minimum norm solution under natural assumptions. At the same time, it avoids any explicit computation of the norm of the linear operator, which represents a significant practical advantage. The method is formulated in Algorithm 3.

Algorithm 3 Two-step inertial algorithm for SCFP

Initialization: Let $\xi_1 > 0$, $\xi_2 > 0$, $\alpha \in (0, 1]$, $0 < \gamma < 1$, $\lambda \in (0, 1 - k_1)$ and x_{-1}, x_0, $x_1 \in H_1$ be arbitrary. We assume that sequences $\{\beta_n\} \subset \left(0, \frac{1}{1+\alpha}\right)$, $\{\epsilon_{1,n}\}$, $\{\epsilon_{2,n}\}$ are all positive sequences such that:

$$\lim_{n\to\infty} \beta_n = 0, \quad \sum_{n=1}^{\infty} \beta_n = \infty, \quad \lim_{n\to\infty} \frac{\epsilon_{1,n}}{\beta_n} = 0, \quad \lim_{n\to\infty} \frac{\epsilon_{2,n}}{\beta_n} = 0.$$

Iterative Steps: Calculate x_{n+1} as follows:
Step 1. Given the current iterates x_{n-1} and x_n $(n \geq 1)$, compute

$$\begin{cases} t_n = x_n + \xi_{1,n} (x_n - x_{n-1}) + \xi_{2,n} (x_{n-1} - x_{n-2}), \\ v_n = \beta_n (1 - \alpha) x_n + (1 - \beta_n) t_n, \\ z_n = v_n - \tau_n A^*(I - T)Av_n, \end{cases}$$

where

$$\tau_n = (1 - k_2) \gamma \frac{\|(I - T)Av_n\|^2}{\|v_n - Sv_n\|^2 + \|A^*(I - T)Av_n\|^2},$$

and

$$\xi_{1,n} = \begin{cases} \min\left\{\xi_1, \dfrac{\epsilon_{1,n}}{\|x_n - x_{n-1}\|}\right\}, & \text{if } x_n \neq x_{n-1}, \\ \xi_1, & \text{otherwise}, \end{cases} \tag{1}$$

and

$$\xi_{2,n} = \begin{cases} \min\left\{\xi_2, \dfrac{\epsilon_{2,n}}{\|x_{n-1} - x_{n-2}\|}\right\}, & \text{if } x_{n-1} \neq x_{n-2}, \\ \xi_2, & \text{otherwise}. \end{cases} \tag{2}$$

If $\|v_n - Sv_n\|^2 + \|A^*(I - T)Av_n\|^2 = 0$, then stop and v_n is a solution to problem (**??**). Otherwise, go to **Step 2**.
Step 2. Compute

$$x_{n+1} = (1 - \lambda) z_n + \lambda S z_n.$$

Let $n := n + 1$ and return to **Step 1**.

A technical ingredient for the convergence analysis is Lemma 6, which provides a contractive-type estimate for relaxed β-demicontractive operators.

Lemma 6. *Let $U\colon H \to H$ be a β-demicontractive operator with $\mathrm{Fix}(U) \neq \emptyset$, and define $U_\lambda = (1 - \lambda) I + \lambda U$ for $\lambda \in (0, 1 - \beta)$. Then, for all $x \in H$ and $u \in \mathrm{Fix}(U)$, it holds that*

$$\|U_\lambda x - u\|^2 \leq \|x - u\|^2 - \lambda (1 - \beta - \lambda) \|(I - U)x\|^2.$$

The convergence analysis is carried out under three natural assumptions: the nonemptiness of the solution set Ω, the demicontractivity and demiclosedness of the operators S and T, and the bounded linearity of the operator A. Under these hypotheses, the main theorem of the chapter establishes strong convergence of the proposed algorithm to the minimum norm solution of the SCFP.

Theorem 13. *Under the above assumptions, the sequence $\{x_n\}$ generated by Algorithm 3 converges strongly to an element $x^* \in \Omega$, where $x^* = P_\Omega(0)$.*

Chapter 5, **Fractal generation with TTP16-PR** ([31], [27]), studies an application of the iterative scheme introduced in Chapter 3 to complex dynamics. The TTP16-PR process is interpreted as an iteration rule in the complex plane, replacing the classical Picard orbit by a two-operator structure. This leads to Julia-type and Mandelbrot-type fractals whose geometry depends on both the parameter c and the control parameters α, β.

The chapter begins by recalling the standard notions of Julia and Mandelbrot sets. For a complex map $f\colon \mathbb{C} \to \mathbb{C}$, the filled Julia [13] set is defined by

$$K(f) = \{x \in \mathbb{C} : \sup_{n\geq 0} |f^n(x)| < \infty\},$$

while the Julia set is its boundary, $J(f) = \partial K(f)$.

Similarly, the Mandelbrot set [22] is classically associated with the quadratic family $f_c(x) = x^2 + c$ and consists of those parameters $c \in \mathbb{C}$ for which the orbit of the origin remains bounded, that is,

$$M = \{c \in \mathbb{C} : \{f_c^n(0)\} \text{ is bounded}\}.$$

In the present chapter, the dynamics are generated by the interaction of two families of mappings, namely $f_c(x) = x^2 + c$ and $g_c(x) = a\,x^m + \frac{b}{x^p} + c$, where $a, b, c \in \mathbb{C}$ and $m, p \geq 2$. The first one is the classical quadratic map, while the second introduces a rational component combining polynomial growth and pole-type behaviour. This two-mapping setting is precisely what allows the TTP16-PR iteration to produce a richer fractal geometry than the one obtained through classical Picard iteration.

The TTP16-PR procedure is then adapted to the complex plane by considering two operators $T\colon C \to C$, $S\colon C \to C$ and by fixing the control parameters α, $\beta \in (0, 1)$. In this context, the iteration takes the form

$$\begin{cases} x_1 \in C, \\ z_n = (1-\beta)\, x_n + \beta\, T x_n, \\ y_n = T((1-\alpha)\, S x_n + \alpha\, z_n), \\ x_{n+1} = T y_n, \end{cases}$$

for all $n \geq 1$. Thus, instead of iterating a single function, the orbit is generated by a nested interaction between two operators, with the parameters α and β controlling the contribution of each stage. This is the basic mechanism used in the numerical generation of fractal sets throughout the chapter.

Because the iteration is built from two mappings and can be arranged in two different configurations, the chapter establishes two distinct escape criteria, depending on the way the two operators are assigned. The first configuration is $Tx = f_c(x)$, $Sx = g_c(x)$, while the second one is $Tx = g_c(x)$, $Sx = f_c(x)$. Since the polynomial and rational terms have different asymptotic behaviour, the escape analysis must be carried out separately for these two situations.

The first main theoretical result provides a sufficient escape criterion for the first configuration.

Theorem 14. *Consider $Tx = f_c(x)$, $Sx = g_c(x)$, where $m \geq 3$, $p \geq 2$, and a, b, $c \in \mathbb{C}$ with $|a| > 1$. If*

$$|x_1| > \max\left\{ |c|, 2, |b|^{\frac{1}{m+p}}, \left(\frac{2 + \alpha\,\beta}{(1-\alpha)\,(|a|-1)} \right)^{\frac{1}{m-2}} \right\},$$

then $|x_n| \to \infty$ as $n \to \infty$.

The second theoretical result gives the analogous escape criterion for the reversed assignment of the two mappings.

Theorem 15. *Consider $Tx = g_c(x)$, $Sx = f_c(x)$, where $m \geq 3$, $p \geq 2$, and a, b, $c \in \mathbb{C}$ with $|a| > 1$. If*

$$|x_1| > \max\left\{ |c|, |b|^{\frac{1}{m+p}}, 2, \left(\frac{3-\alpha}{\alpha\,\beta\,(|a|-1)} \right)^{\frac{1}{m-2}}, \left(\frac{2}{|a|-1} \right)^{\frac{1}{m-1}}, \left(\frac{1}{|a|-1} \right)^{\frac{1}{m-2}} \right\},$$

then $|x_n| \to \infty$ as $n \to \infty$.

These two escape criteria represent the core theoretical contribution of the chapter, as they justify the numerical construction of the corresponding Julia-type and Mandelbrot-type sets generated by TTP16-PR.

Next it is considered the generation of Julia sets by discretizing a region of the complex plane and applying the TTP16-PR iteration to each point. The escape time is used to classify points and approximate the filled Julia set. Two computational procedures are employed, corresponding to the two possible configurations of the operators.

Several parameter configurations are tested in each case, showing that the resulting geometry depends on c, α, β, m, p, a and b. The obtained sets range from compact shapes to more irregular, fragmented, or radially symmetric structures. Moreover, interchanging the roles of the two mappings leads to visibly different dynamical behaviour and geometric patterns.

The chapter then turns to the generation of Mandelbrot-like sets. In this context, the complex parameter c is no longer fixed, but varies over a region of the complex plane, and the initial value is chosen as $x_1 = c$. This choice is natural in the present framework, because the rational term prevents the direct use of the classical initialization $x_1 = 0$. Accordingly, the chapter defines the Mandelbrot-like set associated with the TTP16-PR iteration as

$$M^* = \{c \in \mathbb{C} : \{x_n\} \text{ is bounded}\}.$$

The chapter also considers the generation of Mandelbrot-like sets by varying the complex parameter and applying the TTP16-PR iteration. Two computational procedures are used, corresponding to the two configurations of the operators.

The numerical experiments show that the geometry of the sets depends on all parameters c, α, β, p, a, b and mainly on the degree m. For smaller values, the sets remain close to the classical Mandelbrot shape, while increasing the degree leads to thicker boundaries and more complex, multi-lobed or symmetric structures. In the second configuration, the sets exhibit more pronounced radial and floral patterns, with the number of lobes influenced by m.

All computations are performed with a fixed number of iterations and uniform resolution in both the dynamical and parameter planes. Overall, the results show that the TTP16-PR scheme generates a broad range of fractal structures and extends classical dynamics through its two-operator and multi-step formulation.

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